8.3 $\psi(r) = \frac{1}{\sqrt{\pi}\, a_0^{3/2}} e^{-r/a_0}$

$$P = \int_0^{a_0} 4\pi r^2 |\psi|^2 dr$$

$$= \frac{4\pi}{\pi a_0^3} \int r^2 e^{-2r/a_0} dr$$

$$= \frac{4}{a_0^3} \left[-\frac{a}{4} e^{-2r/a} \left(a^2 + 2ra + 2r^2 \right) \right]_0^{a_0}$$

$$= \left[-e^{-2} (1 + 2 + 2) \right] - \left[e^{-0} \right]$$

$= $ [illegible]

$= $ [illegible]

$$= 1 - 5e^{-2} = 0.323$$

8.7.

The energy appears only in the radial equation.

This is a consequence of the spherical symmetry of the central potential $V(r) = \frac{ke}{r}$.

Note: classically, the orbit radius uniquely defines the Energy

8.12

$n = 5$ of Hydrogen Atom

a.) $\ell = 0, 1, 2, 3, 4$

b.) $m_\ell = -4, -3, \ldots 0, \ldots 3, 4$

c.) $L = \sqrt{\ell(\ell+1)}\,\hbar = 0, \sqrt{2}\,\hbar, \sqrt{6}\,\hbar, 2\sqrt{3}\,\hbar, 2\sqrt{5}\,\hbar$

8.13. Show $\psi = Cre^{-r/2\delta}\cos\theta$ is a solution of:

$$-\frac{\hbar^2}{2m}\left[\frac{P(\theta)F(\phi)}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{R(r)F(\phi)}{r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) + R(r)P(\theta)\frac{d^2F}{d\phi^2}\right]$$

$$-\frac{Ke^2}{r}R(r)P(\theta)F(\phi) = E\,R(r)P(\theta)F(\phi)$$

$$\psi = Cre^{-r/2\delta}\cos\theta$$

$$R(r) = re^{-r/2\delta}$$

$$R(\theta) = C\cos\theta$$

$$F(\phi) = 1$$

$$\frac{dR}{dr} = e^{-r/2\delta} - \frac{r}{2\delta}e^{-r/2\delta} = \frac{R}{r} - \frac{R}{2\delta} = R\left(\frac{1}{r} - \frac{1}{2\delta}\right)$$

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = \frac{d}{dr}\left[R\left(r - \frac{r^2}{2\delta}\right)\right] = \frac{dR}{dr}\left(r - r^2/2\delta\right) + R\left(1 - \frac{r}{\delta}\right)$$

$$= R\left(\frac{1}{r} - \frac{1}{2\delta}\right)\left(r - \frac{r^2}{2\delta}\right) + R\left(1 - \frac{r}{\delta}\right)$$

$$= R\left(1 - \frac{r}{2\delta} - \frac{r}{2\delta} + \frac{r^2}{4\delta^2} + 1 - \frac{r}{\delta}\right) = R\left(2 + \frac{r^2}{4\delta^2} - \frac{2r}{\delta}\right)$$

$$\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) = C\frac{d}{d\theta}\left(-\sin^2\theta\right) = -C\cdot 2\sin\theta\cos\theta = -2\sin\theta\, P$$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\left(2+\frac{r^2}{4\delta^2}-\frac{2r}{\delta}\right)+\frac{1}{r^2\sin\theta}\left(-2\sin\theta\right)\right]-\frac{ke^2}{r}=E$$

$$-\frac{\hbar^2}{2m}\left[\cancel{\frac{2}{r^2}}+\frac{1}{4\delta^2}-\frac{2}{r\delta}-\cancel{\frac{2}{r^2}}\right]-\frac{ke^2}{r}=E$$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{4\delta^2}-\frac{2}{r\delta}\right]-\frac{ke^2}{r}=E$$

is a solution when

$$\frac{\hbar^2}{m\delta}=ke^2 \qquad \delta=\frac{\hbar^2}{ke^2m}$$

$$E=-\frac{\hbar^2}{8m\delta^2}$$

b.) $$E=-\frac{\hbar^2}{8m}\left(\frac{ke^2m}{\hbar^2}\right)^2=-\frac{m(ke^2)^2}{8\hbar^2}=-\frac{\alpha^2mc^2}{2(2^2)}$$

$$n=2$$

$$\alpha=\frac{ke^2}{\hbar c}$$

c.) Since $n=2$, $\ell=0$ or 1.

ψ has θ dependence, so $\ell=1$

$$L=\sqrt{\ell(\ell+1)}\,\hbar=\sqrt{2}\,\hbar$$