Experimental tests of Bell Inequalities

Resolution of E.P.R. Paradox
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

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In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.
EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not ‘Complete’ Even Though ‘Correct.’

SEE FULLER ONE POSSIBLE

Believe a Whole Description of ‘the Physical Reality’ Can Be Provided Eventually.
On the Problem of Hidden Variables in Quantum Mechanics

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The demonstrations of von Neumann and others, that quantum mechanics does not permit a hidden variable interpretation, are reconsidered. It is shown that their essential axioms are unreasonable. It is urged that in further examination of this problem an interesting axiom would be that mutually distant systems are independent of one another.
\[ |0,0\rangle = \frac{1}{\sqrt{2}} \left( |RR\rangle + |LL\rangle \right) \]

The spin components along the direction of motion add to zero, so \( m = 0 \).

The fact that this combination is spin-0, \(|0,0\rangle\), is not obvious. The state \(|1,0\rangle\) has the minus sign.

Being careful about direction of rotation of classical E vector
for particle 1 moves in +x direction and particle 2 moves in -x direction (see hw 5.13)

\[ |L\rangle_1 = \left( |x\rangle + i |y\rangle \right) / \sqrt{2} = |L\rangle_2 \]

\[ |R\rangle_1 = \left( |x\rangle - i |y\rangle \right) / \sqrt{2} = |R\rangle_2 \]

\[ |0,0\rangle = \frac{1}{\sqrt{2}} \left( |x\rangle x \langle y| + |y\rangle y \langle x| \right) \]

see problem 5.23
Take $\hat{a} = x^\dagger$, $b = \cos \theta \ x^\dagger + \sin \theta \ y^\dagger$

The state that is polarized along $\hat{b}$:

$$|b\rangle = b_1 |x\rangle + b_2 |y\rangle$$

Rotate to get state (spin-1 photon)

$c = \cos \theta$, $s = \sin \theta$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|b\rangle = c \cos \theta |x\rangle - s \sin \theta |y\rangle$$

Coupling rate $R(\theta, \phi) = \left| \langle b, x | 0, 0 \rangle \right|^2$

$$\langle b, x | 0, 0 \rangle = \frac{1}{\sqrt{2}} \langle b_1 | x \rangle \langle 0 | 0 \rangle = \frac{1}{\sqrt{2}} \ c \cos \theta$$

$$\langle b, x | 0, 0 \rangle \langle 0, 0 | b, x \rangle = \frac{1}{2} \ c^2 \cos^2 \theta$$
FIG. 4. Normalized coincidence rate as a function of the relative polarizer orientation. Indicated errors are ±1 standard deviation. The solid curve is not a fit to the data but the prediction of quantum mechanics.
Typical coincidence rates without polarizers are 240 coincidences per second in the null delay. The generalized Bell theorem\textsuperscript{2,3} yields the following inequalities:

\[-1 \leq S = \left\{ R(\vec{a}, \vec{b}) - R(\vec{a}, \vec{b}') + R(\vec{a}', \vec{b}) + R(\vec{a}', \vec{b}') - R_1(\vec{a}') - R_2(\vec{b}) \right\} / R_0 \leq 0, \tag{1}\]

where $R(\vec{a}, \vec{b})$ is the rate of coincidences with polarizer I in orientation $\vec{a}$ and polarizer II in orientation $\vec{b}$, $R_1(\vec{a}')$ is the coincidence rate with polarizer II removed and polarizer I in orientation $\vec{a}'$ [and similarly for $R_2(\vec{b})$], and $R_0$ is the coincidence rate with the two polarizers removed. On the other hand,

\[\begin{align*}
(a) & \quad \begin{array}{c}
\vec{a} \\
\vec{b}
\end{array} & \quad (b) & \quad \begin{array}{c}
\vec{a} \\
\vec{b}
\end{array}
\end{align*}\]

FIG. 3. Orientations leading to the maximum violations of generalized Bell inequalities.
Although we never observed any deviation from rotational invariance, we have measured in a special run the quantities involved in $S$ [Eq. (1)] for one particular set of orientations as shown in Fig. 3(a). We found

$$S_{\text{exp}} = 0.126 \pm 0.014,$$

violating inequality (1) by 9 standard deviations and in good agreement with QM prediction $S_{\text{QM}} = 0.118 \pm 0.005$. 
The EPR experiment, special relativity, and the distinction between effects and signals

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For over 50 years the Einstein–Podolsky–Rosen experiment\(^1\) has generated considerable debate among physicists. Much of that discussion has centered on reconciling the instantaneous correlation of measurements over spacelike intervals with the principles of special relativity. Toward this end, some have felt the need to give up the concept of locality, implying that the correlation of measurements does not result from one measurement influencing another. In this note, I shall present an alternate interpretation in which the two measurements of the EPR experiment are considered to affect one another. I hope to show that such an interpretation is conceptually satisfying if viewed in the context of the distinction between “encoded” signals and other physical effects in special relativity.

Consider Bohm’s formulation\(^2\) of the EPR experiment. A source of electrons is placed between two Stern–Gerlach detectors whose measurements are made along the same transverse axis. The source emits pairs of electrons in the singlet state,

\[
|\psi\rangle = (1/\sqrt{2}) \left[ (|+\rangle - |--\rangle) \right],
\]

with the electrons moving in opposite directions, one toward each detector. After trips of arbitrary length (through vacuum), the electrons enter the detectors. Quantum theory predicts that whenever one detector measures the spin of one of the two electrons in the singlet pair as pointing up, the other will measure the spin of its electron as pointing down.

Distinction need not be a troubling one. So long as it is kept in mind that relativity does not forbid nonsignal-carrying effects from traveling at arbitrarily high velocity, the concept of the observations of one of the EPR detectors affecting the measurement of the other is a philosophically tractable one.
Weihs et al. PRL 81 (5039) 1998

Loophole:

1. Inefficient detection
2. Space-like separation of “observer” (not sinusoidal switching like Aspect)

Source = “degenerate” type-II parametric down conversion

\[ |v\rangle = \frac{1}{\sqrt{2}} (|H\rangle |v\rangle - |v\rangle |H\rangle) \]
FIG. 2. One of the two observer stations. A random number generator is driving the electro-optic modulator. Silicon avalanche photodiodes are used as detectors. A “time tag” is stored for each detected photon together with the corresponding random number “0” or “1” and the code for the detector “+” or “−” corresponding to the two outputs of the polarizer.
Light from 250 m length

Generalized Bell inequality:

\[ S(x_1, x_1', y_2, y_2') = |E(x_1, y_2) - E(x_1', y_2')| + |E(x_1, y_2') + E(x_1', y_2)| \leq 2 \]

\[ S_{\text{max}}(0, 45^\circ, 22.5^\circ, 67.5^\circ) = 2\sqrt{2} = 2.82 \]

due to imperfect correlation "visibile?"

of source (97%) expect \( S = 2.74 \)

\[ S_{\text{exp}} = 2.73 \pm 0.02 \]

\( N = 14,700 \)
Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km

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For more than 80 years, the counterintuitive predictions of quantum theory have stimulated debate about the nature of reality1. In his seminal work2, John Bell proved that no theory of nature that obeys locality and realism can reproduce all the predictions of quantum theory. Bell showed that in any local realist theory the correlations between distant measurements satisfy an inequality and, moreover, that this inequality can be violated according to quantum theory. This provided a recipe for experimental tests of the fundamental principles underlying the laws of nature. In the past decades, numerous ingenious Bell inequality tests have been reported3-12. However, because of experimental limitations, all experiments to date required additional assumptions to obtain a contradiction with local realism, resulting in loopholes12-15. Here we report on a Bell experiment that is free of any such additional assumption and thus directly tests the principles underlying Bell’s inequality. We employ an event-ready scheme2,16,17 that enables the generation of high-fidelity entanglement between distant electron spins. Efficient spin readout avoids the fair sampling assumption (detection loophole13,14), while the use of fast random basis selection and readout combined with a spatial separation of 1.3 km ensure the required locality conditions12. We perform 245 trials testing the CHSH-Bell inequality18 $S \leq 2$ and find $S = 2.42 \pm 0.20$. A null hypothesis test yields a probability of $p = 0.039$ that a local-realist model for space-like separated sites produces data with a violation at least as large as observed, even when allowing for memory15,19 in the devices. This result rules out large classes of local realist theories, and paves the way for implementing device-independent quantum-secure communication20 and randomness certification21,22.
Entangled State of B mesons

e^{+}e^{-} \rightarrow \gamma \rightarrow bb \text{ produces } B_0 \overline{B_0} \text{ entangled state. }

\[ \Upsilon \rightarrow \frac{1}{\sqrt{2}} \left[ |B_0 \rangle_1 |\overline{B_0} \rangle_2 - |\overline{B_0} \rangle_1 |B_0 \rangle_2 \right] \]

\[ B_0(\overline{B_0}) \rightarrow e^{\pm} X^{\mp} \]

$B_0 \overline{B_0}$ are completely anti-correlated. A decay of one is a measurement of the state and collapses the wave function.
example BELLE event
BaBar mixing measurement

Fig. 6 shows two distributions, one for the interval $\Delta t$ between the times of decays $B_d \to l^+X$ and $B_{\bar{d}}^{-} \to \psi K_S$ and the other one for the CP conjugate process $B_{\bar{d}}^{-} \to l^-X$ and $B_d \to \psi K_S$. They are clearly different proving that CP is broken.

**Figure 6**: The observed decay time distributions for $B^0$ (red) and $\bar{B}^0$ (blue) decays.

time asymmetry!
difference vs. $\Delta t$

time difference between decays measured as length between decay vertices. to be continued…
“Yet the main point to be noted is that EPR correlations, which represent some of quantum mechanics most puzzling features, serve as an essential precision tool, which is routinely used in these measurements. I feel it is thus inappropriate to refer to EPR correlations as a paradox.” I.I. Bigi

“When you come to a fork in the road, take it.” Yogi Berra

“I know of no more concise formulation of one of quantum mechanics most counter-intuitive features that underlies the interference pattern observed in a double-slit experiment with particle beams: even a single electron can pass through both slits.” I.I. Bigi
Emergent Gravity and the Dark Universe

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Recent theoretical progress indicates that spacetime and gravity emerge together from the entanglement structure of an underlying microscopic theory. These ideas are best understood in Anti-de Sitter space, where they rely on the area law for entanglement entropy. The extension to de Sitter space requires taking into account the entropy and temperature associated with the cosmological horizon. Using insights from string theory, black hole physics and quantum information theory we argue that the positive dark energy leads to a thermal volume law contribution to the entropy that overtakes the area law precisely at the cosmological horizon. Due to the competitive exponents they exhibit, such additional `ble scales: identification of the state series, and usters

Figure 1: Two possible quantum entanglement patterns of de Sitter space with a one-sided horizon. The entanglement between EPR pairs is represented pictorially by tiny ER-bridges. The entanglement is long range and connects bulk excitations that carry the positive dark energy either with the states on the horizon (left) or primarily with each other (right). Both situations leads to a thermal volume law contribution to the entanglement entropy.