Significance of Electromagnetic Potentials in the Quantum Theory

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In this paper, we discuss some interesting properties of the electromagnetic potentials in the quantum domain. We shall show that, contrary to the conclusions of classical mechanics, there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish. We shall then discuss possible experiments to test these conclusions; and, finally, we shall suggest further possible developments in the interpretation of the potentials.
assume this almost everywhere in the following discussions) we have, for the region inside the cage, \( H = H_0 + V(t) \) where \( H_0 \) is the Hamiltonian when the generator is not functioning, and \( V(t) = e\phi(t) \). If \( \psi_0(x,t) \) is a solution of the Hamiltonian \( H_0 \), then the solution for \( H \) will be

\[
\psi = \psi_0 e^{-iS/\hbar}, \quad S = \int V(t) dt,
\]

which follows from

\[
\frac{i\hbar}{\partial t} \left( \frac{\partial \psi_0}{\partial t} + \frac{\partial S}{\partial t} \right) e^{-iS/\hbar} = [H_0 + V(t)]\psi = H\psi.
\]

The new solution differs from the old one just by a phase factor and this corresponds, of course, to no change in any physical result.
EM potential for electron

Aharonov–Bohm

\[ L = -e \phi + \frac{e}{c} \mathbf{A} \cdot \mathbf{v} \]

\[ q = -e \]

long solenoid: \[ \mathbf{B} = B_0 \hat{z} \]

for \( s > R \)

\[ A = B_0 R^2 \frac{s}{2s} \]

\[ \int A^2 \, ds = \frac{B_0 R^2}{2s} (2 \pi s) = B_0 \pi R^2 = \Phi_B \]

circle of constant \( \phi \)
Interference comes from electron taking both paths, just like Young’s double slit.
\[ S_{ce} = \frac{e}{c} \int_{0}^{T} \vec{A} \cdot \vec{v} \, dt = \frac{e}{c} \int_{Path} \vec{A} \cdot d\vec{s} \]

\[ \psi_1 + \psi_2 = e^{\frac{iS_2}{\hbar}} (\psi_1 e^{i(S_1-S_2)/\hbar} + \psi_2) \]

\[ S_1 - S_2 = \frac{e}{c} \int_{Path} \vec{A} \cdot d\vec{s} = \frac{e}{c} \Phi_B \]

\[ P = |\psi_1 + \psi_2|^2 = 1 + \cos\left(\frac{e\Phi_B}{hc}\right) \]

P varies sinusoidally with field strength B.
Such a field was produced by an iron whisker, about 1 μm in diameter and 0.5 mm long, placed in the shadow of the fiber f.
FIG. 2. (a) Fringe pattern due to biprism alone. (b) Pattern displaced by 2.5 fringe widths by field of type $a'$. 

Field $a'$
Field a (whisker) shows interference fringes

(a) Tilted fringes produced by tapering whisker in shadow of biprism fiber. (b) Fresnel fringes in the shadow of the whisker itself, just outside shadow of fiber. (c) Same as (b), but from a different part of the whisker, and with fiber out of the field of view.
Experimental confirmation of Aharonov-Bohm effect using a toroidal magnetic field confined by a superconductor

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The electron holography technique was employed to make a crucial test of the existence of the Aharonov-Bohm (AB) effect. The relative phase shift was measured between two electron waves passing through spaces inside and outside a tiny toroidal ferromagnet, covered completely with a superconductor layer and a Cu layer. Below the transition temperature the relative phase shift was measured to be 0 or π due to the magnetic-flux quantization in units of h/2e. The results directly demonstrated the existence of the AB effect even when the magnetic field was confined by the surrounding superconductor due to the Meissner effect and an electron beam was prevented from penetrating the magnet.
FIG. 1. Conceptual diagram of the experiment. A Cu layer for shielding from an electron wave is not shown.
T > T_c

\[ \Delta \phi = 0 \]

\[ \Delta \phi \text{ not quantized} \]

T < T_c

\[ \Delta \phi = \pi \]

FIG. 10. Interferograms of toroids at 15 and 5 K. (a), (b), and (c): magnetic flux is quantized in \( n (\hbar / 2e) \) (\( n \) is even) below \( T_c \). The toroid is \( R1 \) (see Table I); (d), (e), and (f): magnetic flux is quantized in \( n (\hbar / 2e) \) (\( n \) is odd) below \( T_c \). The toroid is \( R2 \). (a) and (d), \( T = 15 \) K (phase amplification, \( \times 2 \)). (b) and (e), \( T = 5 \) K (phase amplification, \( \times 2 \)). (c) and (f), \( T = 5 \) K (phase amplification, \( \times 1 \)).
Dephasing in electron interference by a ‘which-path’ detector

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Wave–particle duality, as manifest in the two-slit experiment, provides perhaps the most vivid illustration of Bohr’s complementarity principle: wave-like behaviour (interference) occurs only when the different possible paths a particle can take are indistinguishable, even in principle. The introduction of a which-path (welcher Weg) detector for determining the actual path taken by the particle inevitably involved coupling the particle to a measuring environment, which in turn results in dephasing (suppression of interference). In other words, simultaneous observations of wave and particle behaviour is prohibited. Such a manifestation of the complementarity principle was demonstrated recently using a pair of correlated photons, with measurement of one photon being used to determine the path taken by the other and so prevent single-photon interference. Here we report the dephasing effects of a which-path detector on electrons traversing a double-path interferometer. We find that by varying the sensitivity of the detector we can affect the visibility of the oscillatory interference signal, thereby verifying the complementarity principle for fermions.
In our experiment we used a double-path electronic interferometer, fabricated within the plane of a high-mobility two-dimensional electron gas. The two paths are defined by two slits electrons can pass through, with one slit in the form of a coherent quantum dot (QD). The QD is a trap that captures electrons for a relatively long time, like a resonant delay line, thus allowing the electrons to be detected more easily. Near the QD, but electrically separated from it, a quantum point contact (QPC) is fabricated, serving as a which-path detector. The QPC is a short conducting segment with width comparable to the electron wavelength, allowing only a small number of modes to pass. It is expected that an electron passing the QD-slit will interact with the nearby QPC-detector (both systems are thus ‘entangled’) and modify the conductance of the QPC. This detection process leads to dephasing, that is, to a suppression of the double-path Aharanov–Bohm interference.
quantum point contact (QPC) measures e-path with some probability
Aharonov-Bohm effect

Fig. 1a: longer paths, resulting from multiple reflections from walls, are much less probable. A phase difference between the two direct paths, \( \Delta \alpha = 2\pi \Phi / \Phi_0 \), is induced via the Aharonov-Bohm effect (for a review see ref. 10). Here \( \Phi \) is the magnetic flux threaded through the area, \( A \), enclosed by these two paths and \( \Phi_0 = h/e \) is the flux quantum. Consequently, the collector current oscillates as a function of magnetic field \( B \) with a period \( \Delta B = \Phi_0 / A = 2.6 \text{ mT} \), corresponding to a phase difference between the two paths equal to \( 2\pi \), as seen in Fig. 2a.
How certain is our which-path detection? An electron entering the QD-slit changes the transmission probability of the QPC-detector by $\Delta T_d$. The rate at which particles probe the detector at zero temperature is $2eV_d/h$, where $V_d$ is the voltage across the detector. Thus, the number of particles probing the detector

with $V_d = 100 \mu$eV. (a.u., arbitrary units.)

The conductance of the QD, $g_{QD}$, and the transmission of the QPC nearby, $T_d$, as a function of the plunger gate voltage, $V_p$. The inset shows schematically the coupled structures.
interference decreases with determination of which path

remaining oscillatory component squared leads to the peak-to-valley value. Last, the visibility is found by dividing by the average value of $I_C$. Error bars indicate the fluctuations in visibility due to fluctuations of device’s properties (instrumental noise is negligibly small). \textbf{c}, The visibility of the AB conductance oscillations as a function of $V_d$ for a fixed $T_d = 0.2$. \textbf{The behaviour is linear for } $eV_d > k_B \Theta$ with saturation for low $V_d$. $=7\mu eV$