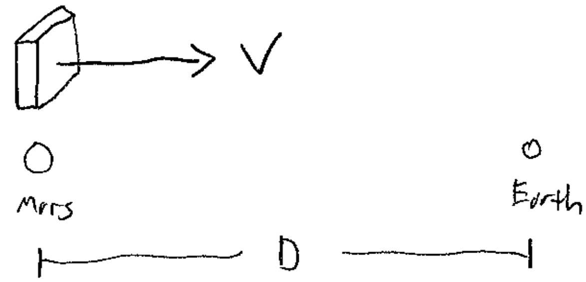
Exam 2 Solutions

(1.)



$D = 10^8$ km

$= 10^{11}$ m

the ship is length contracted in the direction of travel

$$V_{observed} = (L)(L)\left(\frac{L}{\gamma}\right) = V_0/\gamma$$

$$\gamma = \frac{V_{observed}}{V_0} = 2$$

$$\frac{1}{\sqrt{1 - v^2/c^2}} = 2$$

$$1 - v^2/c^2 = \frac{1}{4}$$

$$\boxed{v = c\,\frac{\sqrt{3}}{2}}$$

Travel time in Earth's frame

is $\Delta t = \frac{D}{V}$

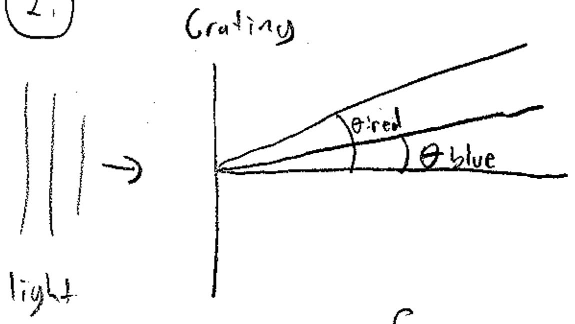
Travel time in Borg's frame is

$$\Delta t_{Borg} = \frac{\Delta t}{\gamma} = \frac{D}{\gamma V} = \frac{10^{11}\,m}{2(\sqrt{3}/2)(3 \times 10^8\, m/s)}$$

$$\boxed{\Delta t_{Borg} = \frac{\sqrt{3}}{9} \times 10^3\, s}$$

$$\approx 2 \times 10^2\, s$$

(2.)



$d \sin\theta = m\lambda$

$m = 1$ for first order

for small angles

$\sin\theta \approx \theta$

$\pi/100$ is small, so θ_{red}, θ_{blue} are small

$$\theta_{red} - \theta_{blue} = \frac{\lambda_{red}}{d} - \frac{\lambda_{blue}}{d} = \pi/100$$

$$d = \frac{100(\lambda_{red} - \lambda_{blue})}{\pi}$$

$$\text{lines per mm} = \frac{1\,mm}{d}$$

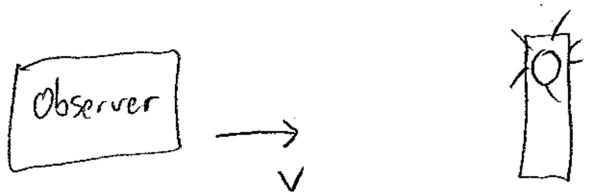
$$= \frac{\pi(10^{-3}m)}{100(650\,nm - 450\,nm)}$$

$$= \frac{\pi(10^{-3}m)}{100(2 \times 10^{-7})}$$

$$= \boxed{\frac{\pi}{2} \times 10^{2}}$$

$$\approx 150$$

(3)



from doppler shift

$$f\lambda = c$$

$$f = f_0 \left(\frac{1 + v/c}{1 - v/c} \right)^{1/2}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_0} \left(\frac{1 + v/c}{1 - v/c} \right)^{1/2}$$

$$\left(1 + v/c\right) = \left(\frac{\lambda_0}{\lambda}\right)^2 \left(1 - v/c\right)$$

$$v/c \left(1 + \left(\frac{\lambda_0}{\lambda}\right)^2\right) = \left(\frac{\lambda_0}{\lambda}\right)^2 - 1$$

$$v = c \, \frac{\left(\frac{\lambda_0}{\lambda}\right)^2 - 1}{1 + \left(\frac{\lambda_0}{\lambda}\right)^2}$$

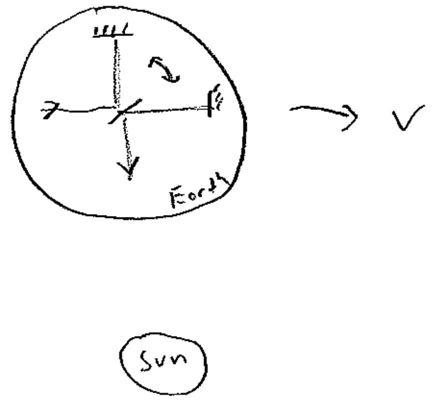
$$\boxed{V = c \, \frac{\left(\frac{650}{550}\right)^2 - 1}{1 + \left(\frac{650}{550}\right)^2}}$$ ← full credit

$$= c \, \frac{\left(\frac{13}{11}\right)^2 - 1}{\left(\frac{13}{11}\right)^2 + 1} \approx c \, \frac{1.2^2 - 1}{1.2^2 + 1} \approx c \, \frac{1.4 - 1}{1.4 + 1} \approx \frac{0.4}{2.4} c$$

$$\approx \frac{1}{6} c$$

actual : $0.17\,c$

(4.)

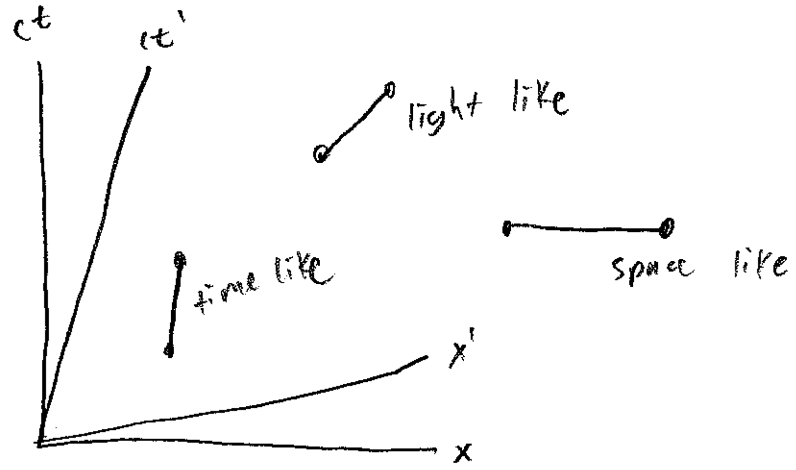


The Michelson-Morley Experiment tested the theory that light progates via an 'Ether'.

From the Ether theory, light traveling in the two arms of an interferometer would take diffrent lengths of time depending on the orientation of the Interferometer. Rotating the interferometer would give fringe shifts.

The null result helped to disprove the Ether theory.

(5.)



(6.) Rayleigh's Criterion applies to diffraction from a circular Aperature.

Two objects are barely resolved when the maximum of the diffraction pattern from the first object is at the first minimum of the second.