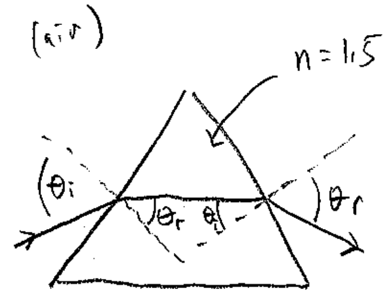
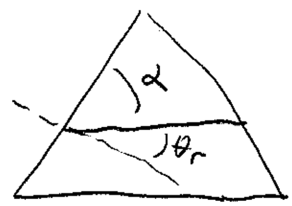
262 Final Exam Solutions

(1.)

a.)



b.) Entering the prism:



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_i = (1.5)\sin\theta_r$$

$$\alpha = 60^\circ \qquad \theta_r = 30^\circ$$

$$\sin\theta_i = (1.5)(\sin 30^\circ) = \frac{3}{4}$$

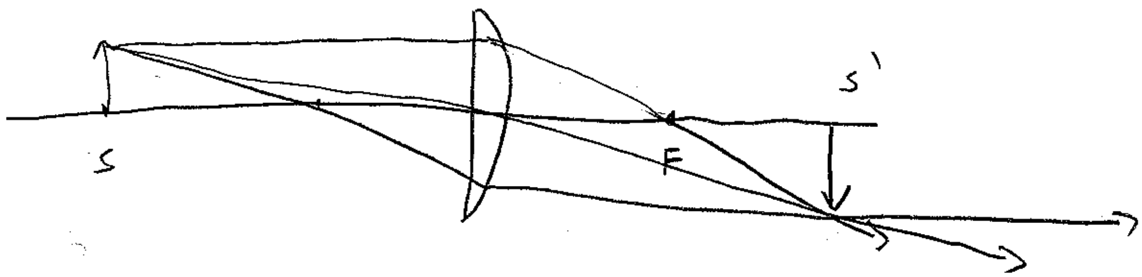
$$\theta_i = \sin^{-1}\left[3/4\right]$$

Exiting the prism

$$\theta_i = 30^\circ$$

$$\theta_r = \sin^{-1}\left[3/4\right)$$

2.



$$m = -\frac{s'}{s}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$s = 2f$$

$$\frac{1}{2f} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{2f}$$

$$s' = 2f$$

$$m = \frac{-s'}{s} = -1$$

(3.)

① ② $\longrightarrow$ $0.5c$

a.) $p = \gamma m v$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.5)^2}} = \frac{1}{\sqrt{\frac{3}{4}}}$$

$$\gamma = \frac{2}{\sqrt{3}}$$

$$P_1 = 0$$

$$P_2 = \frac{2}{\sqrt{3}} m(0.5c) = \frac{m}{\sqrt{3}} c$$

$$P_1 + P_2 = \frac{m}{\sqrt{3}} c$$

b.) $E = \gamma m c^2$

$$E_1 = mc^2$$

$$E_2 = \gamma m c^2$$

$$E_1 + E_2 = (\gamma + 1) mc^2 = \left(1 + \frac{2}{\sqrt{3}}\right) mc^2$$

ℓ

c.)

① ② $\longrightarrow$ in S

$\leftarrow$ ① ② $\rightarrow$ in S'. S' moves with velocity V,

find velocities of each mass in S' and equate the magnitudes (speeds)

mass 1:

$$V_x' = \frac{V_x - V}{1 - \frac{V_x V}{c^2}} = \frac{0 - V}{1 - 0} = -V$$

mass 2:

$$V_x' = \frac{0.5c - V}{1 - \frac{(0.5c)V}{c^2}} = \frac{\frac{c}{2} - V}{1 - \frac{V}{2c}}$$

In my case S' move in the positive x-direction, so V is positive.

equating speeds:

$$|-V| = \left| \frac{\frac{c}{2} - V}{1 - \frac{V}{2c}} \right|$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$V = \frac{\frac{c}{2} - V}{1 - \frac{V}{2c}}$$

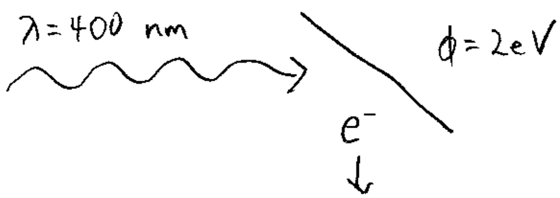
use quadratic formula

$$V - \frac{V^2}{2c} = \frac{c}{2} - V \qquad \frac{V^2}{2c} - 2V + \frac{c}{2} = 0.$$

$$V = \frac{2 \pm \sqrt{4 - 1}}{\frac{1}{c}} \qquad \boxed{V = (2 - \sqrt{3})c}$$

$V = (2 + \sqrt{3})c > c$ $\leftarrow$ not physical.

4.



a.) $$E_{photon} = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{400 \text{ nm}} \sim 3 \text{ eV}$$

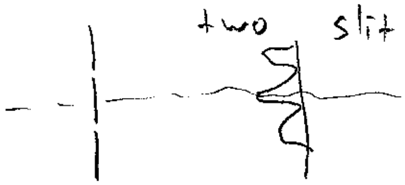
$$E_{e^-} = E_{photon} - \phi \sim 1 \text{ eV}$$

for electron

$$E = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2}$$

$$\lambda = \frac{hc}{\sqrt{2E\, mc^2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\left(2(1 \text{eV})(0.511 \times 10^6 \text{eV})\right)^{1/2}} \sim 1 \text{ nm}$$

b.) two slit interference



first minima at $d\sin\theta = \pm \lambda/2$

$$\theta = \pm \sin^{-1}\left[\frac{\lambda}{2d}\right]$$

angular width (minima to minima)

is then

$$2\theta = 2\sin^{-1}\left[\frac{\lambda}{2d}\right]$$

(5.) $\tau = 2\ \mu s = 2 \times 10^{-6}\ s$

accelerated to $0.8c$.

observed lifetime will be longer from time dilation

$$\tau_{obs} = \tau_0 \gamma$$

$$\gamma = \frac{1}{\left(1 - (0.8)^2\right)^{1/2}} = \frac{1}{\sqrt{.36}} = \frac{1}{.6} = \frac{5}{3}$$

$$\tau_{obs} = \frac{5}{3} \times 2\ \mu s = 3.3\ \mu s$$

(6.)

$\psi(x) = Ae^{ikx}$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{2x^2} + U(x)\psi = E\psi$$

for a free particle, $U(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\frac{d\psi}{dx} = (ik)\psi$$

$$\frac{d^2\psi}{dx^2} = (ik)^2\psi = -k^2\psi$$

$$-\frac{\hbar^2}{2m}\left(-k^2\psi\right) = E\psi$$

$$E = \frac{\hbar^2 k^2}{2m}$$

(7.)

$B = 1\mu T$

$E = Bc = (1\times10^{-6}\ T)(3\times10^{8}\ m/s)$

$= 300\ Tm/s = 300\ \frac{V}{m}$

$I = \frac{1}{2}\varepsilon_0 c E_{max}^2 = \frac{E_{max} B_{max}}{2\mu_0}$

$\mu_0 = 4\pi \times10^{-7}\ N\cdot A^{-2}$

$= \frac{(300\ \frac{V}{m})(1\times10^{-6}\ T)}{2(4\pi\times10^{-7}\ N\cdot A^{-2})} \approx \frac{3\times10^{-4}}{3\times10^{-6}}\ \frac{W}{m^2} = 100\ \frac{W}{m^2}$

(8.) For Hydrogen $E_n = \frac{-13.6\ eV}{n^2}$

for one electron Helium $E_n = \frac{-(Z)^2\ 13.6\ eV}{n^2}$

$\Delta E = \frac{hc}{\lambda} = Z^2(-13.6\ eV)\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 3\,(13.6\ eV)$

$\lambda = \frac{hc}{\Delta E} = \frac{(1240\ eV\cdot nm)}{3(13.6\ eV)} \sim 30\ nm$

(9.) +10 points for attempting the final.

(Exam points added to only 90 points)