Physics 262 HW # 13 Solutions

(1.)

show $(\Delta s)^2 = (\Delta s')^2$ use

$$x' = \gamma(x - vt)$$
$$ct' = \gamma\left(ct - \frac{v}{c}x\right)$$
$$y' = y$$
$$z' = z$$

$$(\Delta s')^2 = (ct')^2 - (x')^2 - (y')^2 - (z')^2$$

$$= \gamma^2\left[\left(ct - \frac{v}{c}x\right)^2 - \left(x - \frac{v}{c}ct\right)^2\right] - y^2 - z^2$$

$$= \gamma^2\left[(ct)^2 + \frac{v^2}{c^2}x^2 - \cancel{2vxt} - x^2 - \frac{v^2}{c^2}(ct)^2 + \cancel{2vxt}\right] - y^2 - z^2$$

$$= \gamma^2\left[(ct)^2\left(1 - \frac{v^2}{c^2}\right) - x^2\left(1 - \frac{v^2}{c^2}\right)\right] - y^2 - z^2$$

$$= (ct)^2 - x^2 - y^2 - z^2 = (\Delta s)^2$$

(2.)

show $(1+\varepsilon)^N \approx 1 + N\varepsilon$ when $\varepsilon \ll 1$, $N = 2, 3$

$N = 2$

$$(1+\varepsilon)^2 = 1 + 2\varepsilon + \varepsilon^2$$

very small

$$\approx 1 + 2\varepsilon$$

$N = 3$

$$(1+\varepsilon)^3 = 1 + 3\varepsilon + 3\varepsilon^2 + \varepsilon^3$$

very small

$$\approx 1 + 3\varepsilon$$

For any N, make a Taylor series expantion around $\varepsilon = 0$

$$f(\varepsilon) = (1+\varepsilon)^N$$

$$f(\varepsilon) = f(0) + \frac{f'(0)\varepsilon}{1!} + \frac{f''(0)\varepsilon^2}{2!} + \cdots$$

$$= 1 + N\varepsilon + \frac{N(N-1)\varepsilon^2}{2}$$

very small

$$\approx 1 + N\varepsilon$$

(3.) Ohanian 2.13

$$x' = \gamma(x - Vt)$$

$$ct' = \gamma\left(ct - \frac{Vx}{c}\right)$$

Solving for unprimed variables

$$\left(\frac{V}{c}\right)x' = \left(\frac{V}{c}\right)\gamma(x - Vt) \qquad \text{eliminate } x$$

$$+ \quad ct' = \gamma\left(ct - \frac{Vx}{c}\right)$$

$$\left(\frac{V}{c}\right)x' + ct' = \left(-\frac{V^2 ct}{c^2} + ct\right)\gamma$$

$$ct\left(1 - \frac{V^2}{c^2}\right) = \frac{1}{\gamma}\left(\frac{Vx'}{c} + ct'\right)$$

$$\boxed{ct = \frac{\left(\frac{Vx'}{c} + ct'\right)}{\gamma\left(1 - V^2/c^2\right)}} \qquad (1)$$

$$cx' = c\gamma(x - Vt) \qquad \text{eliminate } t$$

$$+ \quad Vct' = V\gamma\left(ct - \frac{Vx}{c}\right)$$

$$cx' + Vct' = \gamma\left(cx - \frac{V^2 x}{c}\right) = \gamma x c\left(1 - \frac{V^2}{c^2}\right)$$

$$\boxed{x = \frac{(x' + Vt')}{\gamma\left(1 - \frac{V^2}{c^2}\right)}} \qquad (2)$$

by reversing prime $\leftrightarrow$ unprime

$V \leftrightarrow -V$

$$x = \gamma\left(x' + Vt'\right) \qquad (3)$$

$$ct = \gamma\left(ct' + \frac{Vx'}{c}\right) \qquad (4)$$

using (1) and (4)

$$\gamma\left(ct' + \frac{Vx'}{c}\right) = \frac{1}{\gamma}\,\frac{\left(ct' + \frac{Vx'}{c}\right)}{\left(1 + \frac{V^2}{c^2}\right)}$$

$$\gamma^2 = \frac{1}{1 - \frac{V^2}{c^2}}$$

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}}$$

using (2) and (3)

$$\gamma\left(x' + Vt'\right) = \frac{1}{\gamma}\,\frac{\left(x' + Vt'\right)}{1 - V^2/c^2}$$

$$\gamma^2 = \frac{1}{1 - \frac{V^2}{c^2}}$$

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}}$$

(4.) Ohanian 2.20

two events seperated by

$\Delta t = 8.0\ s$

$\Delta x = 2.0 \times 10^9\ m$

Calculate the invariant, $(\Delta s)^2$

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$= \left[(3.0\times10^8\ \tfrac{m}{s})(8.0\ s)\right]^2 - \left[2.0\times10^9 m\right]^2$$

$$(\Delta s)^2 = 1.8\times10^{18}\ m^2$$

this is a time-like interval,

so <u>events cannot be made simultaneous.</u>

Since the separation is time-like, a spaceship could travel between the two events with $v < c$. - they can be made to happen at same point in space in spaceship frame.

$\boxed{\Delta x' = 0}$

$$(c\Delta t')^2 = (\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$(\Delta t')^2 = (\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2 = (8.0\ s)^2 - \left(\frac{2.0\times10^9\ m}{3.0\times10^8\ m/s}\right)^2$$

$$\boxed{\Delta t' = 4.4\ s}$$

(5.) Ohanian 2.22.

$$V_1 < C$$
$$V_2 < C$$

show $$\frac{(V_1 + V_2)}{1 + \frac{V_1 V_2}{c^2}} < C$$

$$\frac{V_1}{c} + \frac{V_2}{c} < 1 + \frac{V_1 V_2}{c^2}$$

$$0 < 1 - \frac{V_1}{c} - \frac{V_2}{c} + \frac{V_1 V_2}{c}$$

$$0 < \left(1 - \frac{V_1}{c}\right)\left(1 - \frac{V_2}{c}\right)$$

each term is always positive when $V_1 < C$, $V_2 < C$.

Other Solutions may be possible __

(6.)

(3.6)

$$o \xrightarrow{\text{plane}} \quad V_1 = 300 \text{ m/s} \quad + 300 \text{ m/s}$$

$$o \xrightarrow[\text{earth}]{} \quad V_2 = 300 \text{ m/s}$$

following treatment leading to Ohanian eq. 38

$$\Delta\tau_2 - \Delta\tau_1 = t\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right) = \frac{t}{2c^2}\left(V_1^2 - V_2^2\right)$$

$$= \frac{t}{2c^2}\left(V_1 - V_2\right)\left(V_1 + V_2\right)$$

$$= \frac{L\,(V_1 + V_2)}{2c^2}$$

$$= \frac{(8 \times 10^6 \text{ m})(9 \times 10^2 \text{ m/s})}{2\,(3 \times 10^8 \text{ m/s})^2} = 4 \times 10^{-8} \text{ s}$$

will also accept treating the earth as an inertial ref. frame.

then, from Ohanian eq. 14

$$\Delta\tau - \Delta t = -VL/2c^2$$

$$= \frac{(3 \times 10^2 \text{ m/s})(8 \times 10^6 \text{ m})}{2\,(3 \times 10^8 \text{ m/s})^2} = 1 \times 10^{-8} \text{ s}$$