1. Stella sees light that is doppler shifted. Outward journey:

\[ f = f_0 \left( \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{\frac{1}{2}} \]

Inward journey:

\[ f = f_0 \left( \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{\frac{1}{2}} \]

In Stella's frame, it takes \( \frac{L}{V} \) for each leg of the journey. She intercepts:

\[ N = \left( \frac{L}{V} \right) f_0 \left( \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{\frac{1}{2}} + \left( \frac{L}{V} \right) f_0 \left( \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{\frac{1}{2}} \]

Stella calculates Terra's age to be:

\[ N T_0 = \frac{N}{f_0} \]

\[ \Delta t_{\text{terra}} = \frac{L}{V} \left[ \left( \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{\frac{1}{2}} + \left( \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{\frac{1}{2}} \right] = \frac{L}{V} \left[ \frac{(1 - \frac{v}{c}) + (1 + \frac{v}{c})}{1 - \frac{v^2}{c^2}} \right] \]

\[ \Delta t_{\text{terra}} = \frac{2L}{V} \]

The same as Terra would calculate.
When Terra leaves Earth, the age difference is
\[ \Delta t_1 = (1 \text{ y})(1 - \frac{1}{8}) = 1 \text{ year} \left(1 - (1 - 0.4^2)^{\frac{1}{2}}\right) \]
\[ \Delta t_1 = 0.0835 \text{ y}. \]

In Earth's reference frame, they will meet when
\[ L_0 + (0.4c) t = (0.8c) t \]

\[ (0.4c) t = (1 \text{ y})(c) \]

\[ t = 2.5 \text{ y} \]

The age difference after Terra leaves Earth is
\[ \Delta t_2 = 2.5 \text{ y} \left(\frac{1}{Y_{\text{Ter}} - \frac{1}{Y_{\text{St}}}}\right) = 2.5 \text{ y} \left((1 - 0.8^2)^{\frac{1}{2}} - (1 - 0.4^2)^{\frac{1}{2}}\right) \]
\[ = -0.3165 \]

\[ \Delta t_1 + \Delta t_2 = -0.23 \]

Sella older than Terra.
In the mirror's rest frame, reflected light will have some frequency of incident light.

In mirror frame:

\[ f = f_0 \left( \frac{1 + \gamma c}{1 - \gamma c} \right)^{1/2} \]

Transform back to original rest frame, light is approaching:

\[ f_{\text{obs}} = f \left( \frac{1 + \gamma c}{1 - \gamma c} \right)^{1/2} = f_0 \frac{1 + \gamma c}{1 - \gamma c} \]
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\[ V = 0.5 \text{c} \]

\[ y = 1.15 \]

Area = \[ 2(1\text{m})^2 + 4(1\text{m}, \frac{1\text{m}}{y}) = 5.5 \text{m}^2 \]

Volume = \[ (1\text{m})(1\text{m})(\frac{1\text{m}}{y}) = 0.87 \text{m}^3 \]
a.) Earth's ref. frame

\[ \gamma_1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ \Delta x = L \left( \frac{1}{\gamma_1} + \frac{1}{\gamma_1} \right) \]

\[ \Delta x = 2Vt - vt \]

\[ t = \frac{L}{v} \left( \sqrt{1 + \frac{v^2}{c^2}} + \sqrt{1 - \frac{v^2}{c^2}} \right) \]

b.)

\[ v_2 = \frac{2V - V}{1 - 2V^2} = \frac{V}{1 - 2V^2} \]

\[ \Delta x = v_2 t = \frac{L}{\gamma_2} + L \]

\[ \gamma_2 = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \left( \frac{1 - \frac{v_2^2}{c^2}}{1 - \frac{v^2}{c_2}} \right)^{\frac{1}{2}} \]

\[ t = \frac{L}{\gamma_2} \left( \frac{1}{\gamma_2} + 1 \right) = \frac{L}{V} \left( 1 - \frac{v_2^2}{c^2} \right) \left( 1 + \frac{1 - 5V^2 + 4V^4}{1 - 2V^2} \right) \]

\[ t = \frac{L}{V} \left( 1 - \frac{v_2^2}{c^2} + \sqrt{1 - 5V^2 - 4V^4} \right) \]
c.)
\[ \frac{t_1}{t_2} = \frac{1}{\frac{1}{8} - 1} \]
\[ v_1 = \frac{V - 2v}{1 - 2v^2/c^2} = \frac{V}{1 - 2v^2/c^2} \]

Same as in b)

\[ t = \frac{1}{v_1} \left( \frac{1}{8} + 1 \right) \]
\[ v_1 = -v_2 \]

\[ t_{\text{Earth}} < t_{\text{Ship}} \text{ by time dilation} \]

faster passing in ship's frame,