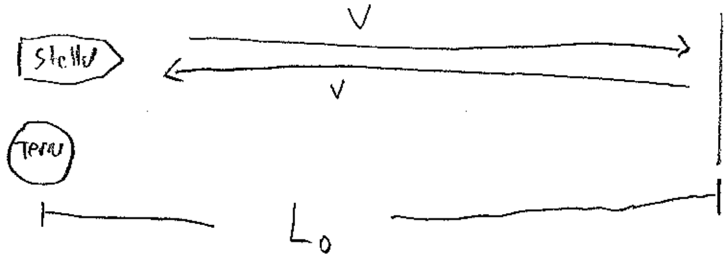
HW #14 Solutions

(1.)




Terra emits light with frequency $f_0$.

Stella sees light that is doppler shifted.

outward journey

$$f = f_0 \left( \frac{1 - v/c}{1 + v/c} \right)^{1/2}$$

inward journey

$$f = f_0 \left( \frac{1 + v/c}{1 - v/c} \right)^{1/2}$$

In Stella's frame, it takes $\frac{L}{\gamma V}$ for each leg of the journey.

she intercepts

$$N = \left(\frac{L}{\gamma V}\right) f_0 \left( \frac{1 - v/c}{1 + v/c} \right)^{1/2} + \left(\frac{L}{\gamma V}\right) f_0 \left( \frac{1 + v/c}{1 - v/c} \right)^{1/2}$$

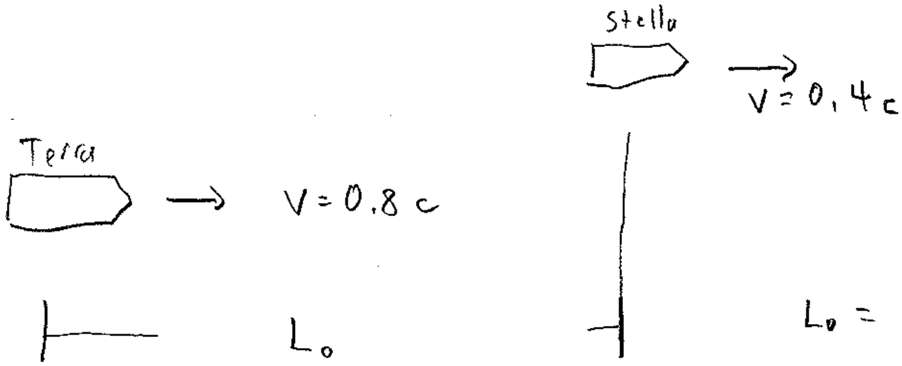
wave fronts

Stella calculates Terra's age to be $N T_0 = \frac{N}{f_0}$

$$\Delta t_{terra} = \frac{L}{\gamma V}\left[ \left( \frac{1 - v/c}{1 + v/c} \right)^{1/2} + \left( \frac{1 + v/c}{1 - v/c} \right)^{1/2} \right] = \frac{L}{V\cancel{\gamma}}\left[ \frac{(1 - \cancel{v/c}) + (1 + \cancel{v/c})}{\cancel{1 - v^2/c^2}} \right]$$

$$\Delta t_{terra} = \frac{2L}{V}$$ ↖ the same as Terra would calculate.

(2.) Ohanian 3.12




$$L_o = (1\,y)(0.4c)$$

when Terra leaves earth, the age difference is

$$\Delta t_1 = (1\,y)\left(1 - \frac{1}{\gamma}\right) = 1\,\text{year}\left(1 - (1 - 0.4^2)^{\frac{1}{2}}\right)$$

$$\Delta t_1 = .0835\ y.$$

In earth's reference frame, they will meet when

$$L_o + (0.4c)t = (0.8c)t$$

$$(0.4c)\,t = (1\,y)(c)$$

$$t = 2.5\ y$$

the age difference after terra leaves earth is

$$\Delta t_2 = 2.5\,y\left(\frac{1}{\gamma_{Terra}} - \frac{1}{\gamma_{Stella}}\right) = 2.5\,y\left((1 - 0.8^2)^{\frac{1}{2}} - (1 - (0.4)^2)^{\frac{1}{2}}\right)$$

$$= -0.3165$$

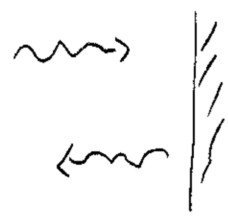
$$\Delta t_1 + \Delta t_2 = -0.23$$

Stella older than terra

(3) Ohanian 3.19


In the mirror's Ref. frame, reflected light will have same frequency of incident light

In mirror frame

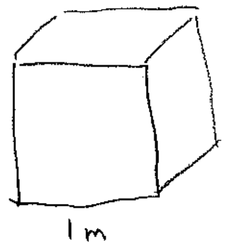


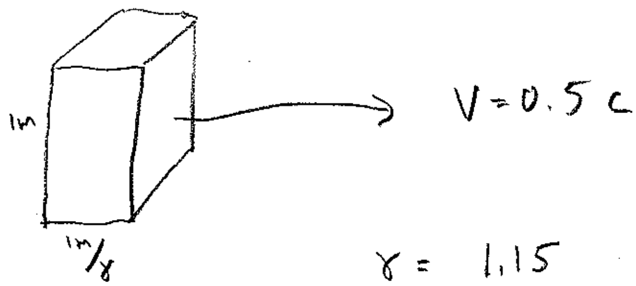
$$f = f_0 \left( \frac{1 + v/c}{1 - v/c} \right)^{1/2}$$

transform back to original Ref. frame, light is approaching,

$$f_{obs} = f \left( \frac{1 + v/c}{1 - v/c} \right)^{1/2} = f_0 \, \frac{1 + v/c}{1 - v/c}$$

(4.) Ohanian 3.27




$\gamma = 1.15$

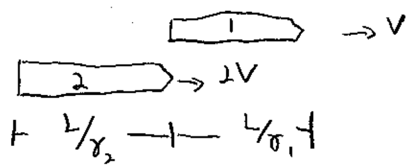
$$\text{Area} = 2(1\,m)^2 + 4\left(1m \cdot \frac{1m}{\gamma}\right) = 5.5\ m^2$$

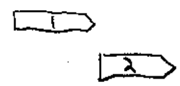
$$\text{Volume} = (1m)(1m)\left(\frac{1m}{\gamma}\right) = 0.87\ m^3$$

(5.) Ohanian 3.32

a.) Earth's Ref. frame

starting to pass



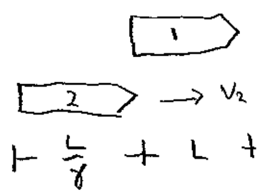

finished passing

$$\Delta x = L\left(\frac{1}{\gamma_2} + \frac{1}{\gamma_1}\right) \qquad \gamma_1 = \frac{1}{\left(1 - v^2/c^2\right)^{1/2}}$$

$$\Delta x = 2Vt - Vt \qquad \gamma_2 = \frac{1}{\left(1 - 4v^2/c^2\right)^{1/2}}$$

$$t = \frac{L}{V}\left(\sqrt{1 - 4v^2/c^2} + \sqrt{1 - v^2/c^2}\right)$$

b.)



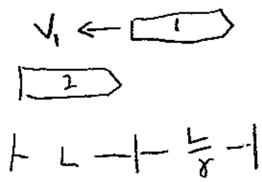

$$V_2 = \frac{2V - V}{1 - 2V^2/c^2} = \frac{V}{1 - 2V^2/c^2}$$

$$\Delta x = V_2 t = \frac{L}{\gamma} + L \qquad \gamma = \frac{1}{\left(1 - V_2^2/c^2\right)^{1/2}} = \frac{1}{\left(1 - \frac{V^2/c^2}{\left(1 - 2V^2/c^2\right)^2}\right)^{1/2}}$$

$$t = \frac{L}{V_2}\left(\frac{1}{\gamma} + 1\right) = \frac{L}{V}\left(1 - 2V^2/c^2\right)\left(1 + \frac{\sqrt{1 - 5V^2/c^2 + 4V^4/c^4}}{\left(1 - 2V^2/c^2\right)}\right)$$

$$t = \frac{L}{V}\left(1 - 2V^2/c^2 + \sqrt{1 - 5V^2/c^2 + 4V^4/c^4}\right)$$

c.)




$$V_1 = \frac{V - 2V}{1 - 2V^2/c^2} = -\frac{V}{1 - 2V^2/c^2}$$

Same as in b.)

$$t = -\frac{L}{V_1}\left(\frac{1}{\gamma} + 1\right) \qquad V_1 = -V_2$$

$t_{earth} < t_{ship}$ by time dilation

faster passing in ship's frame,