Physics 262 HW # 15 Solutions

(1.) Ohanian 4.6

$v = 250$ km/s

Kinetic Energy

$K_R = (\gamma - 1) mc^2$ ← Relativistic

$K_N = \frac{1}{2} mv^2$ ← Newtonian

$\gamma = (1 - v^2/c^2)^{-1/2}$ use Taylor Series Expansion

$$\gamma = 1 + \frac{v^2}{2c^2} + \frac{-\frac{1}{2}(-\frac{1}{2}-1)}{2}\left(\frac{v^2}{c^2}\right)^2 = 1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots$$

$$K_R = (\gamma - 1)mc^2 \approx \left(\frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4}\right)mc^2 = \frac{1}{2}mv^2 + \frac{3}{8}\frac{v^4 m}{c^2}$$

fractional difference

$$\frac{K_R - K_N}{K_N} = \frac{\frac{3}{8} m v^4/c^2}{\frac{1}{2} m v^2} = \frac{3}{4}\frac{v^2}{c^2}$$

$$= \frac{3}{4}\left(\frac{250 \times 10^3 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right)^2 = \boxed{5.2 \times 10^{-7}}$$

Momentum

$P_R = \gamma m v$

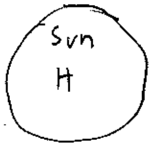
$P_N = m v$

fractional difference

$$\frac{P_R - P_N}{P_N} = \frac{\gamma m v - m v}{m v} = \gamma - 1$$

$$\approx \frac{v^2}{2c^2} = \frac{1}{2}\left(\frac{250 \times 10^3 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right) = \boxed{3.5 \times 10^{-7}}$$

(2.) Ohanian 4.13



$$T_H = 1.5 \times 10^7 \text{ K}$$

$$E_{thermal} = N * \frac{3}{2} kT$$

$$N = \frac{N_A}{A} \times 10^3 \text{ g}$$

↖ grams per mole

$$E_{thermal} = \frac{3 N_A kT}{2A} \times 10^3 \text{g} = \frac{3(6.02 \times 10^{23} \text{mole}^{-1})(1.38 \times 10^{-23} \text{ J/K})(1.5 \times 10^7 \text{ K})}{2(1 \text{ g/mole})} \times 10^3 \text{g}$$

$$\boxed{E_{thermal} = 1.9 \times 10^{11} \text{ J}}$$

$$M_{thermal} = \frac{E_{thermal}}{c^2} = \boxed{2.1 \times 10^{-6} \text{ Kg}}$$

$$M_{thermal} \ll m_{rest}$$

$$\frac{M_{thermal}}{m_{rest}} \times 100 = 2.1 \times 10^{-4} \text{ percent increase}$$

N

(3.) Ohanian 6.6.

$$R = 0.529 \times 10^{-10}\ m$$

$$v = \frac{c}{137}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{R^2} = \frac{1}{4\pi(8.9\times10^{-12}\ F/m)}\frac{1.6\times10^{-19}\ C}{(5.29\times10^{-11}\ m)^2}$$

$$\boxed{E = 5.2\times10^{11}\ V/m}$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel}$$

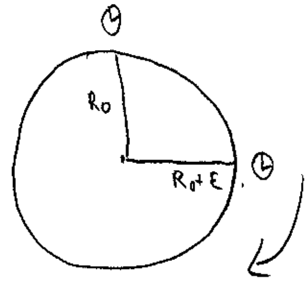
$$\vec{B}'_{\perp} = \gamma\vec{B}_{\perp} + \frac{\gamma}{c^2}\vec{E}\times\vec{V}$$

$$\vec{B} = 0$$

$$B' = \frac{\gamma}{c^2}Ev = \frac{\gamma e v}{4\pi\varepsilon_0 c^2 R^2} = \frac{(1.6\times10^{-19}\ C)\left(\frac{1}{137}\right)(3\times10^{8}\ m/s)}{\left(1-\frac{1}{137^2}\right)4\pi(8.9\times10^{-12}\ F/m)(3\times10^{8}\ m/s)^2(5.29\times10^{-11}\ m)^2}$$

$$\boxed{B' = 12\ T}$$

(4.) Ohanian 7.7



$v = 460$ m/s

$\varepsilon = 21$ km

Special Relativity time dilation factor is

$$\frac{1}{\gamma}$$

G.R time dilation is

$$1 + \frac{gh}{c^2}$$

$$\frac{1}{\gamma} - \left(1 + \frac{gh}{c^2}\right) \approx 1 + \frac{v^2}{2c^2} - \left(1 + gh/c^2\right)$$

$$= \frac{v^2}{2c^2} - gh/c^2$$

$$= \frac{1}{c^2}\left(\frac{v^2}{2} - gh\right)$$

$$= \frac{1}{(3 \times 10^8 \text{ m/s})^2}\left(\frac{(460 \text{ m/s})^2}{2} - (9.8 \text{ m/s}^2)(21 \times 10^3 \text{ m})\right)$$

$$= -1 \times 10^{-12}$$

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(5)

Show $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

$$\vec{A} = \hat{i}\, a_x + \hat{j}\, a_y + \hat{k}\, a_z$$

$$\vec{B} = \hat{i}\, b_x + \hat{j}\, b_y + \hat{k}\, b_z$$

$$\vec{C} = \hat{i}\, c_x + \hat{j}\, c_y + \hat{k}\, c_z$$

first

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \Big(\hat{i}(b_y c_z - b_z c_y) + \hat{j}(b_z c_x - b_x c_z) + \hat{k}(b_x c_y - b_y c_x) \Big)$$

$$= \hat{i}\Big[a_y(b_x c_y - b_y c_x) - a_z(b_z c_x - b_x c_z) \Big]$$

$$+ \hat{j}\Big[a_z(b_y c_z - b_z c_y) - a_x(b_x c_y - b_y c_x) \Big]$$

$$+ \hat{k}\Big[a_x(b_z c_x - b_x c_z) - a_y(b_y c_z - b_z c_y) \Big]$$

$$= \hat{i}\Big[b_x(a_y c_y + a_z c_z) - c_x(a_y b_y + a_z b_z) \Big]$$

$$+ \hat{j}\Big[b_y(a_x c_x + a_y c_y) - c_y(a_x b_x + a_y c_y) \Big]$$

$$+ \hat{k}\Big[b_z(a_x c_x + a_y c_y) - c_z(a_x b_x + a_y b_y) \Big]$$

(5) cont.

we can add missing terms - ei

$$\hat{i}\left[b_x(a_y c_y + a_z c_z) - c_x(a_y b_y + a_z b_z)\right]$$

$$= \hat{i}\left[b_x(a_x c_x + a_y c_y + a_z c_z) - c_x(a_x b_x + a_y b_y + a_z b_z)\right]$$

these add to zero

$\vec{A}\cdot\vec{C}$

$\vec{A}\cdot\vec{B}$

we can do this for all missing terms to give

$$\vec{A}\times(\vec{B}\times\vec{C}) = \hat{i}\left[b_x\, \vec{A}\cdot\vec{C} - c_x\, \vec{A}\cdot\vec{B}\right]$$

$$+ \hat{j}\left[b_y\, \vec{A}\cdot\vec{C} - c_y\, \vec{A}\cdot\vec{B}\right]$$

$$+ \hat{k}\left[b_z\, \vec{A}\cdot\vec{C} - c_z\, \vec{A}\cdot\vec{B}\right]$$

$$= (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$