Physics 262 HW # 17 Solutions

(1.) Simple Approximation: Use instantaneous Power and

Calculate $P = \frac{\Delta E}{\Delta t}$

$$\Delta t = \frac{\Delta E}{P} \qquad E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Starting from Bohr radius $r = a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2}$

ending at $r = 1 \times 10^{-15}$ m ← approximate size of the nucleus.

$$\Delta E = -\frac{e^2}{8\pi\varepsilon_0}\left[\frac{\pi m_e e^2}{\varepsilon_0 h^2} - 10^{15}\,\text{m}\right] = 1.2 \times 10^{-13}\ \text{J}.$$

$$P = \frac{e^2 a^2}{6\pi\varepsilon_0 c^3} \qquad ma = \frac{v^2}{r} m = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad a = \frac{e^2}{m_e 4\pi\varepsilon_0 r^2}$$

$$\boxed{P = \left(\frac{e^2}{6\pi\varepsilon_0 c^3}\right)\left(\frac{e^2}{4\pi\varepsilon_0 m_e}\right)^2 \frac{1}{r^4}} \qquad (1)$$

using $r = a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{\varepsilon_0 h^2}{\pi m_e e^2}$

$$P = \frac{e^{14} m_e^2 \pi}{96\, \varepsilon_0^7 c^3 h^8} = 4.8 \times 10^{-8}\ \text{J/s}$$

$$\Delta t = \frac{\Delta E}{\Delta t} = 2.4 \times 10^{-6}\ \text{s}.$$

A Better Approximation: Assume circular, stable orbits as electron orbit decays.

→ cont.

(1.) cont.

$$P = \left(\frac{e^6}{96 \pi^3 \varepsilon_0^3 m_e^2 c^3} \right) \frac{1}{r^4}$$

$$E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

negative since Atom is losing energy.

$$P = -\frac{dE}{dt}$$

$$\left(\frac{e^6}{96 \pi^3 \varepsilon_0^3 m_e^2 c^3} \right) \frac{1}{r^4} = -\left(\frac{e^2}{8\pi\varepsilon_0} \right) \left(\frac{1}{r^2} \right) \frac{dr}{dt}$$

$$r^2 dr = -\left(\frac{e^4}{12 \pi^2 \varepsilon_0^2 m_e^2 c^3} \right) dt$$

$$\frac{r^3}{3} = -(\quad) t + C$$

$$r^3 = -\left(\frac{e^4}{4 \pi^2 \varepsilon_0^2 m_e^2 c^3} \right) t + a_0^3$$

constant set to give correct initial conditions.

we can take $r \to 0$

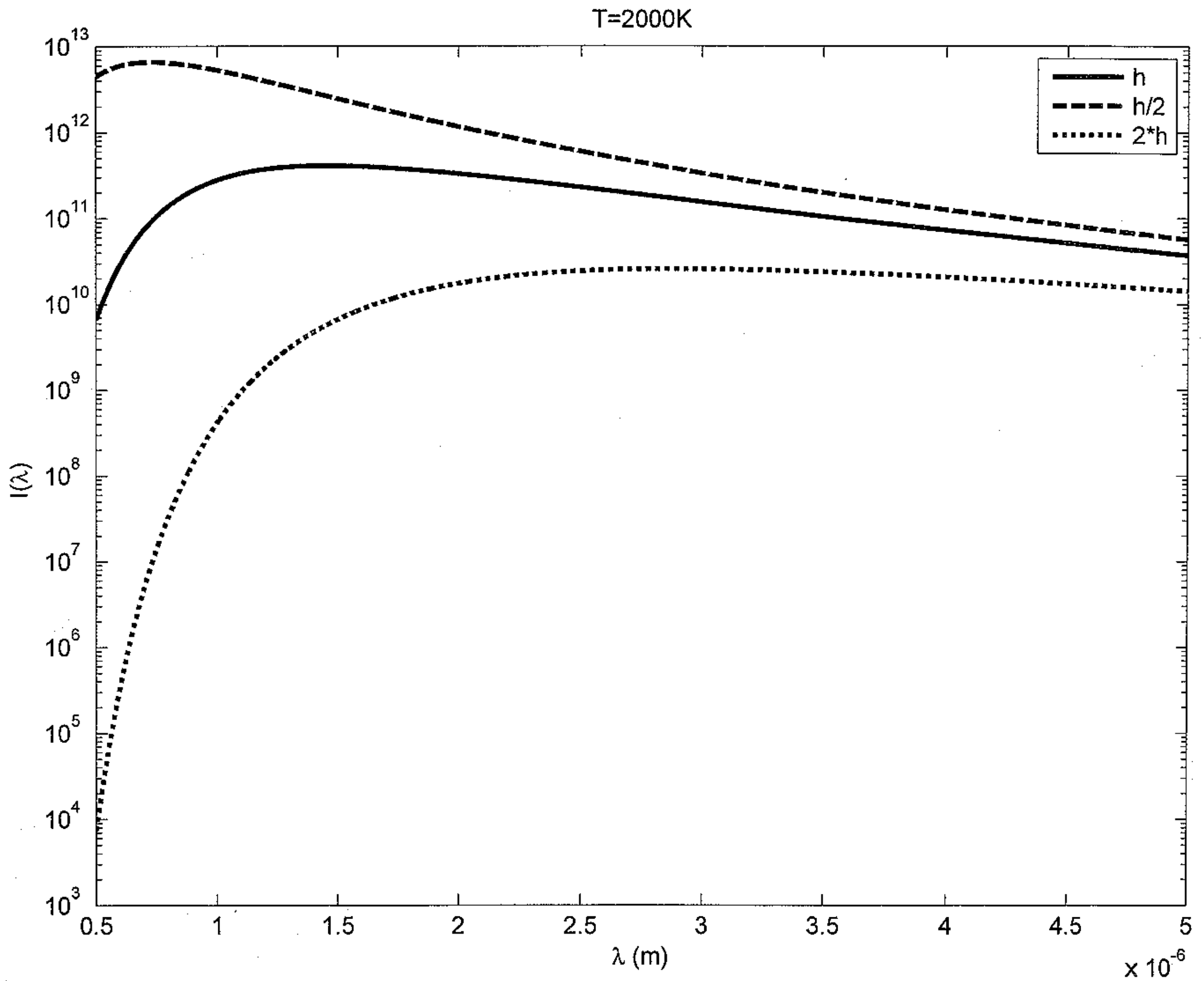
$$t = a_0^3 \left(\frac{4 \pi \varepsilon_0^2 m_e^2 c^3}{e^4} \right)$$

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m_e e^2}$$

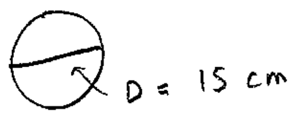
$$t = \frac{4 \varepsilon_0^5 h^6 c^3}{\pi^2 m_e e^{10}} = 5 \times 10^{-12} \text{ s}$$

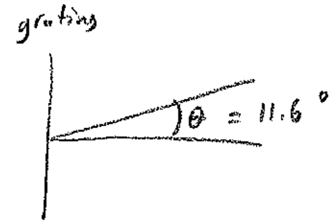
(2.)

any value of T is O.K.



(3.) YF 38.66





a.) $\lambda_m T = 2.9 \times 10^{-3}\ \mathrm{m \cdot K}$ ← wien displacement law

for a diffraction grating $d \sin\theta = m\lambda$

$m = 1$ for first bright fringe from central fringe

$$d = \frac{1}{3850\ \mathrm{cm^{-1}}} = \frac{1}{3850}\ \mathrm{cm} = \frac{1}{3.85 \times 10^{5}}\ \mathrm{m}$$

$$d \sin\theta = \lambda = \frac{2.9 \times 10^{-3}\ \mathrm{m \cdot K}}{T}$$

$$T = \frac{(2.9 \times 10^{-3}\ \mathrm{m \cdot K})}{\left(\frac{1}{38.5}\ \mathrm{m}\right)\left(\sin(11.6^\circ)\right)} = 5550\ \mathrm{K}.$$

b.) $P = \sigma T^4 \left(4\pi \frac{D^2}{2^2}\right)$

$E = Pt$

$$t = \frac{E}{P} = \frac{12.0 \times 10^{6}\ \mathrm{J}}{\left(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\right)(5550)^4 (\pi)(0.15\ \mathrm{m})^2} = 3.15\ \mathrm{s}$$

4.

YF 39.39

a.) $\lambda = 0.10 \times 10^{-6}$ m $=$ 100 nm

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{100 \text{ nm}} = 12.4 \text{ eV}$$

b.) $$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = eV$$

$$E = \frac{(1240 \text{ eV}\cdot\text{nm})^2}{2(0.511 \text{ MeV})(100 \text{ nm})^2} = 1.5 \times 10^{-4} \text{ eV}$$

$$\boxed{1.5 \times 10^{-4} \text{ V}}$$

$$mv = \frac{h}{\lambda} \qquad v = \frac{h}{\lambda m} = \frac{(hc)\,c}{\lambda\, m/c^2} = \frac{(1240 \text{ eV}\cdot\text{nm})(3.0\times 10^8 \text{ m/s})}{(100 \text{ nm})(0.511 \text{ MeV})}$$

$$\boxed{v = 7.3 \times 10^3 \text{ m/s}}$$

c.) $M_p = 938$ MeV/c^2

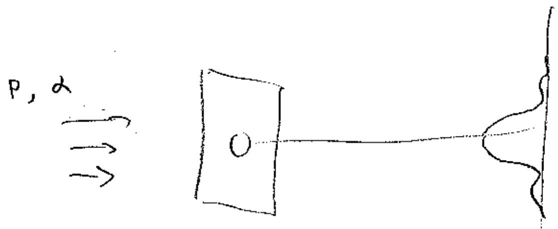
$$E = \frac{(1240 \text{ eV})^2}{2(938 \text{ MeV})(100 \text{ nm})^2} = 8.2 \times 10^{-8} \text{ eV}$$

$$\boxed{V = 8.2 \times 10^{-8} \text{ V}}$$

$$v = \frac{(1240 \text{ eV}\cdot\text{nm})(3.0\times 10^8 \text{ m/s})}{(100 \text{ nm})(938 \times 10^6 \text{ eV})} = 4.0 \text{ m/s}$$

(5.)

39.42



$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

for diffraction from a circular aperture

$$\sin\theta_1 = 1.22\,\lambda/D \qquad \theta_p = 15^\circ$$

$$E_p = eV$$

$$E_\alpha = 2eV$$

$$\frac{h^2}{2m_\alpha \lambda_\alpha^2} = 2\,\frac{h^2}{2m_p \lambda_p^2}$$

$$\lambda_\alpha = \lambda_p \sqrt{\frac{m_p}{2m_\alpha}} = \left(\frac{D\sin\theta_p}{1.22}\right)\sqrt{\frac{m_p}{2m_\alpha}}$$

$$\sin\theta_\alpha = \frac{1.22\,\lambda_\alpha}{D} = \sin\theta_p \sqrt{\frac{m_p}{2m_\alpha}}$$

$$\theta_\alpha = \sin^{-1}\left[\sin\theta_p \sqrt{\frac{m_p}{2m_\alpha}}\right] = \sin^{-1}\left[\sin(15^\circ)\sqrt{\frac{1.67}{2(6.64)}}\right]$$

$$= 5.3^\circ$$

(6.)

39.56

$$E = 2.58 \text{ eV}$$

a.) $E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV}\cdot\text{nm}}{2.58 \text{ eV}} = 480 \text{ nm}$$

b.) $\Delta E \, \Delta t \geq \hbar$

$$\Delta t = 1.64 \times 10^{-7} \text{ s}$$

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{6.68 \times 10^{-16} \text{ eV}\cdot\text{s}}{1.64 \times 10^{-7} \text{ s}} = 4.1 \times 10^{-8} \text{ eV}$$

c.) $E = \frac{hc}{\lambda}$

$$dE = -\frac{hc}{\lambda^2} \, d\lambda$$

$dE \to \Delta E$
$d\lambda \to \Delta\lambda$

$$\frac{|\Delta E|}{|E|} = \frac{\left(\frac{hc}{\lambda^2} \Delta\lambda\right)}{\frac{hc}{\lambda}} = \frac{|\Delta\lambda|}{|\lambda|}$$

$$\Delta\lambda = \frac{\Delta E}{E} \lambda = \left(\frac{\hbar}{\Delta t}\right)\left(\frac{hc}{E^2}\right) = \frac{(6.68 \times 10^{-16} \text{ eV}\cdot\text{s})}{(1.64 \times 10^{-7} \text{ s})} \frac{(1240 \text{ eV}\cdot\text{nm})}{(2.58 \text{ eV})^2} = 7.6 \times 10^{-7} \text{ nm}$$