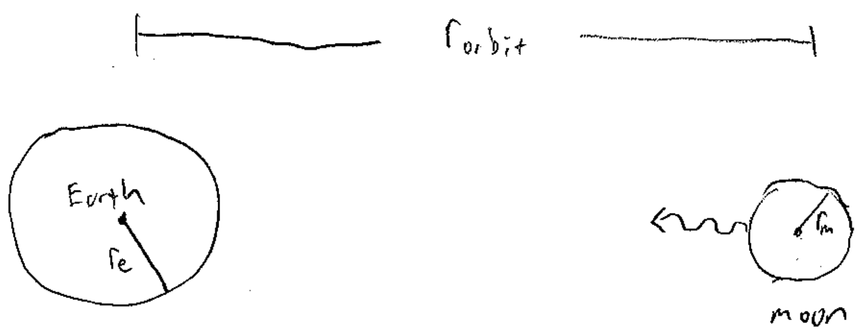
Physics 267 Problem # 10 Solutions



$m_e = 6 \times 10^{24}$ kg

$r_e = 6400$ km

$m_m = 7.4 \times 10^{22}$ kg

$r_m = 1700$ km

$$r_{orbit} = 400{,}000 \text{ km}$$

$$r_{orbit} \gg r_m$$

$$r_{orbit} \gg r_e$$

Assume we can treat the problem as

if $r_{orbit} \to \infty$

from moon to 'space', there is a

red-shift

f_m: moon

f_s: space

using
$$\frac{d\tau_2}{d\tau_1} = 1 + \frac{\Phi_2}{c^2} - \frac{\Phi_1}{c^2}$$

and λ instead of f

$$\frac{\lambda_s}{\lambda_m} = 1 + \cancel{\frac{\Phi_s}{c^2}}^{\,0} - \frac{\Phi_m}{c^2}$$

$$\Phi = \frac{-GM}{r}$$

$r \to \infty$ in space

$$\frac{\lambda_s}{\lambda_m} = 1 + \frac{GM}{r_m c^2}$$

from 'Space' to earth

$$\frac{\lambda_s}{\lambda_e} = 1 + \frac{GM_e}{r_e}$$

$$\lambda_e = \frac{\lambda_s}{\left(1 + \frac{GM_e}{r_e}\right)} = \lambda_m \frac{\left(1 + \frac{GM_m}{r_m c^2}\right)}{\left(1 + \frac{GM_e}{r_e c^2}\right)}$$

when $\frac{GM}{rc^2}$ are small, this is approximately the same as

$$\lambda_e = \lambda_m \left(1 + \frac{GM_m}{r_m c^2} - \frac{GM_e}{r_e c^2}\right)$$

Let's calculate wavelength shift

$$\frac{\lambda_e - \lambda_m}{\lambda_m} = \frac{GM_m}{r_m c^2} - \frac{GM_e}{r_e c^2}$$

$$= \frac{\left(6.7 \times 10^{-11}\ m^3/kg\,s^2\right)}{\left(3 \times 10^8\ m/s\right)^2}\left(\frac{7.4 \times 10^{22}\ kg}{1.7 \times 10^6\ m} - \frac{6 \times 10^{24}\ kg}{6.4 \times 10^6\ m}\right)$$

$$= -7 \times 10^{-10}$$