Physics 267 Problem #13 Solutions

1. photon $\quad E = \dfrac{hc}{\lambda}$

particle with mass $\quad E = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$

photon : $\left(\dfrac{1240 \text{ eV}\cdot\text{nm}}{1 \text{ nm}}\right) = 1240 \text{ eV}$

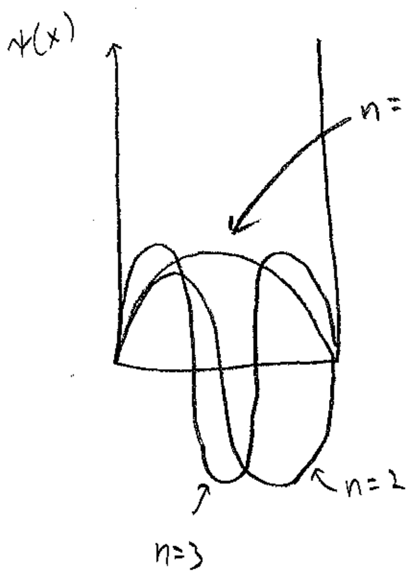
electron : $\dfrac{(hc)^2}{2m_e/c^2\, \lambda^2} = \dfrac{(1240 \text{ eV nm})^2}{2(0.511 \text{ MeV})(1\text{nm})^2} = 1.5 \text{ eV}$

proton : $\dfrac{(1240 \text{ eV nm})^2}{2(938 \text{ MeV})(1\text{nm})^2} = 8.2\times10^{-4} \text{ eV}$

human : $M = 100$ kg

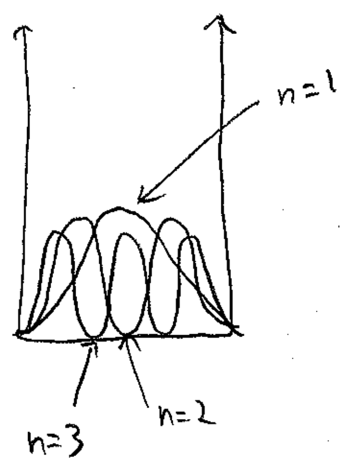
$$\frac{h^2}{2m\lambda^2} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})^2}{2(100 \text{ kg})(1\times10^{-9}\text{m})^2} \times \frac{1 \text{ eV}}{1.6\times10^{-19} \text{ J}} = 1.4\times10^{-30} \text{ eV}.$$

2.



$p(x) = |\psi(x)|^2$

3.



4.

$\psi(x) = \sqrt{\frac{2}{L}} \sin(kx)$

Inside the box $\psi(x)$ looks like a solution that is a combination of free particle wave functions with $p = \hbar k$ and $p = -\hbar k$

where $E = \frac{\hbar^2 k^2}{2m}$ $\boxed{k = \frac{n\pi}{L}}$

$$\psi(x) = \sqrt{\frac{2}{L}} \frac{1}{2i}\left[e^{ikx} - e^{-ikx} \right]$$

check consistency with the uncertainty principle

$$\Delta p_x = \sqrt{\langle p_x^2\rangle - \cancel{\langle p_x\rangle^2}^{\,0 \text{ by symmetry}}}$$

$$\Delta p_x = \sqrt{\langle p_x^2\rangle} = \sqrt{E\, 2m} = \sqrt{\hbar^2 k^2} = \hbar k = \hbar \frac{n\pi}{L}$$

take $\Delta x = L$

$$\Delta x \Delta p_x = \hbar n \pi > \hbar$$