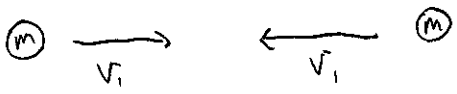
Physics 267 Problem # 9 solutions.

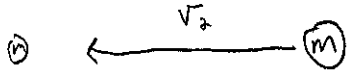


$$E^2 - p^2c^2 = m^2c^4$$

$$E = \gamma_1 mc^2 \qquad p = \gamma_1 m v \qquad \gamma_1 = \frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}}$$

$$E_{total} = 2\gamma_1 mc^2 \qquad p_{total} = 0$$

$$E^2 - p^2c^2 = \boxed{4\gamma_1^2 m^2 c^4}$$



$$v_2 = \frac{-v_1 - v_1}{1 + \frac{v_1^2}{c^2}} = \frac{-2v_1}{1 + v_1^2/c^2}$$

$$E_{total} = mc^2 + \gamma_2 mc^2 \qquad \gamma_2 = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$p_{total} = -\gamma_2 m v_2$$

$$E^2 - p^2c^2 = m^2c^4(\gamma_2 + 1)^2 - \gamma_2^2 m^2 c^2 v_2^2$$

$$= m^2c^4\left(\gamma_2^2 + 2\gamma_2 + 1 - \gamma_2^2 \frac{v_2^2}{c^2}\right)$$

$$= m^2c^4\left[\gamma_2^2\left[1 - \frac{v_2^2}{c^2}\right] + 2\gamma_2 + 1\right] = 2m^2c^4(\gamma_2 + 1)$$

$$\gamma^{-2} = 1 - v_2^2/c^2 = 1 - \frac{4v_1^2/c^2}{\left(1+v_1^2/c^2\right)^2} = \frac{1 - 2v_1^2/c^2 + v_1^4/c^4}{(\quad)^2} = \frac{\left(1-v_1^2/c^2\right)^2}{\left(1+v_1^2/c^2\right)^2}$$

$$\gamma_2 = \gamma_1^2\left(1+v_1^2/c^2\right)$$

$$E^2 - p^2 = 2m^2c^4(\gamma_2 + 1)$$

$$= 2m^2c^4\left(\gamma_1^2\left(1+v_1^2/c^2\right) + 1\right)$$

$$= 2m^2c^4\left(\frac{1+v_1^2/c^2}{1-v_1^2/c^2} + 1\right) = 2m^2c^4\left(\frac{1+1}{1-v_1^2/c^2}\right)$$

$$= \boxed{4m^2c^4\gamma_1^2}$$