Exam 1 solutions

1.


a.) particle 1 : $E_k = 0$

$p = 0$

particle 2 : $E_k = (\gamma - 1) mc^2 = \left( \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right) mc^2$

$p = \gamma m v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mv$

b.) In ref where total $p$ is zero

$E^2 = (0)^2 + (mc^2)^2$ ← invariant

calculate the invariant in the first frame

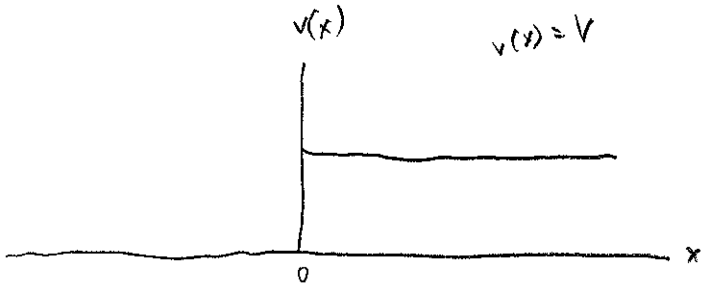
$(mc^2)^2 = E^2 - (pc)^2$

$= \left[ \gamma_1 mc^2 + \gamma_2 mc^2 \right]^2 - (\gamma_2 mvc)^2$

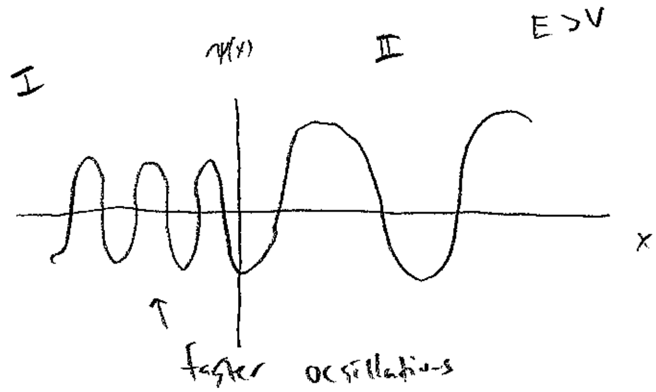
$$E_{first} = \left( \left[ mc^2 + \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mc^2 \right]^2 - \left( \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mvc \right)^2 \right)^{1/2}$$

(2.)

a.)




b.)




c.) Yes, reflection is possible.

Set up possible wave functions in each region

Particle is incident from the right, but can be reflected, so in region II

$$\psi_{II}(x) = C e^{-ik_2 x} + D e^{ik_2 x}$$

$$\psi_{I}(x) = A e^{-ik_1 x} + \cancel{B e^{ik_1 x}} \to 0$$ no particles incident from left

match $\psi_I(0) = \psi_{II}(0)$

$\frac{d\psi_I(0)}{dx} = \frac{d\psi_{II}(0)}{dx}$ to get D in terms of C

$$R = \frac{k_2}{k_1} \frac{|D|^2}{|C|^2}$$

should not be squared in actual ground state wave function

3.)

$$\psi(x) = A e^{-m\omega^2 x^2/2\hbar}$$

for Harmonic Oscillator

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + \frac{1}{2} m\omega^2 x^2 \psi(x) = E\psi(x)$$

find $\frac{d^2\psi(x)}{dx^2}$

$$\frac{d\psi(x)}{dx} = -\frac{\cancel{2} m\omega^2 x}{\cancel{2}\hbar} \psi(x)$$

$$\frac{d^2\psi(x)}{dx} = -\frac{m\omega^2}{\hbar}\psi(x) + \frac{m^2\omega^4}{\hbar^2} x^2 \psi(x)$$

Sub into Schr. eq., cancel $\psi(x)$

$$-\frac{\hbar^2}{2m}\left[-\frac{m\omega^2}{\hbar} + \frac{m^2\omega^4}{\hbar^2}x^2\right] + \frac{1}{2} m\omega^2 x^2 = E$$

$x^2$ terms must cancel, so

$$E = \frac{1}{2}\hbar\omega^2$$ ← check. should not be squared

(4.)

Electrons can lose energy to Exciting the first excited state, but only if they have enough energy.

ground to first excited state in H

is $13.6 \text{ eV} \left[ \frac{1}{1^2} - \frac{1}{2^2} \right]$

$$\Delta E = 13.6 \text{ eV} \cdot \frac{3}{4}$$

when $Ve = \Delta E$, electrons are left with no $E_K$, and can't make past barrier potential and contribute to current.

$I$ peaks then at multiples of $\frac{1}{e}(13.6 \text{ eV})\left(\frac{3}{4}\right)$