Physics 330, Spring 2009
Exam 2
100 points

1. **Problem 1 (30 points)** Consider the hyperfine splitting in the Hydrogen atom. The transition from the higher energy state to the lower energy state gives a 21 cm wavelength photon.
   (a) (10 points) At the temperature of the cosmic microwave background, 2.7 K, what is the population ratio of higher energy state to the lower energy state?
   (b) (15 points) What is the effective magnetic field that produces the energy level splitting? Express your answer in terms of the wavelength given above, fundamental constants and the Bohr magnetron.
   (c) (5 points) What is the origin of this effective B-field?

2. **Problem 2 (30 points)** The Li atom has $Z = 3$.
   (a) (20 points) Neglecting any fine-structure level splitting (assume energy levels are determined only by n), give all possible j values for the first excited state of the neutral Li atom.
   (b) (10 points) Give an example of a transition from the first excited state to the ground state that is forbidden. Explain what selection rule is being violated.

3. **Problem 3 (20 points)**
   (a) (10 points) Sketch the Fermi-Dirac distribution, $f_{FD}(E)$, versus E at $T = 0$ K.
   (b) (10 points) What is Bose-Einstein condensation?

4. **Problem 4 (20 points)** KBr has a equilibrium spacing of $r_0 = 0.3$ nm and a vibrational frequency of $f = 7 \times 10^{12}$ Hz. Calculate the spacing between the two lowest rotational energy levels. The masses of K and Br are m=39 u and m=80 u, respectively. u=1.7×10^{-27} Kg.

(1.)

$T = 2.7\ K$

$hc = 1240\ eV\cdot nm$

a.) $E = \frac{hc}{\lambda}$

$= 1.24 \times 10^{-4}\ eV\cdot cm$

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-(E_2 - E_1)/kT}$$

$$\frac{N_2}{N_1} = e^{-\left(\frac{hc}{\lambda}\right)/kT}$$

$$\frac{N_2}{N_1} = e^{-\left(hc/21\,cm\right)/k(2.7\ K)}$$

estimate : at $T = 300\ K$, $kT \sim \frac{1}{40}\ eV$

at $T = 2.7\ K$, $kT \sim \frac{1}{4000}\ eV \sim .00026\ eV$

$$\frac{hc}{\lambda} = \frac{1.24 \times 10^{-4}\ eV\cdot cm}{21\ cm} \sim 6 \times 10^{-5}\ eV$$

$$\left(\frac{hc}{\lambda}\right)/kT \cong \frac{6 \times 10^{-5}}{2.6 \times 10^{-4}} \sim 0.3$$

$e^{-0.3}$ ← significant population of N_2

b.) $\Delta U = 2\mu_B B_z$

for the electron, $\mu_z = -m_s g_s \mu_B$, $m_s = \pm 1$

$\Delta U = 2\mu_B B$ $g_s \sim 2$

$$B = \frac{\Delta U}{2\mu_B} = \frac{hc}{2\lambda\mu_B} = \frac{hc}{2(21\ cm)\,\mu_B}$$

c.) proton spin

(2.) $Z=3$. Ground state : $1s^2 2s^1$

a.) first excited state is outer electron in $n = 3$ shell.

inner electrons combine to give $J=0$, so total angular momentum is from outer electron.

for $n=3$, $l=0,1,2$
$s=\frac{1}{2}$

j: $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$

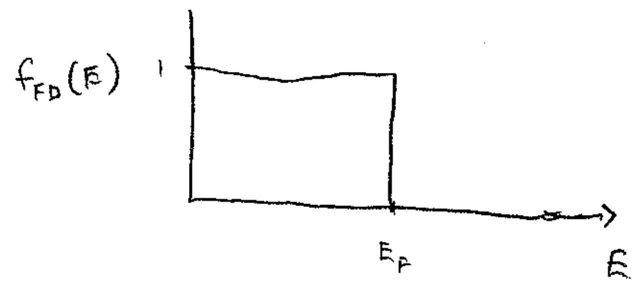
b.) $3S_{1/2} \rightarrow 2S_{1/2}$ $\Delta l = \pm 1$

$3P_{5/2} \rightarrow 2S_{1/2}$ $\Delta j = \pm 1, 0$

$3P_{3/2} \rightarrow 2P_{3/2}$ $\Delta l \pm 1$ (although not to ground state)

3.

a.) $$f_{FD} = \frac{1}{e^{(E-E)/kT} + 1}$$



b.) For Bose particles (bosons) there is no restriction on the number of particles allowed in a certain quantum state.
As $T \rightarrow 0$, all particles "condense" in to the ground state.

(4.)

$r_0 = 0.3$ nm

$f = 7 \times 10^{12}$ Hz

$m_K = 39\ u$

$m_{Br} = 80\ u$

$u = 1.7 \times 10^{-27}$ Kg

$$E_r = \frac{L^2}{2I} \qquad I = u\, r_0^2 = \frac{m_K m_{Br}}{m_K + m_{Br}} r_0^2$$

$$L^2 = \ell(\ell+1)\hbar^2$$

$$E_r = \frac{L^2}{2 r_0^2 \left(\frac{m_K m_{Br}}{m_K + m_{Br}}\right)} = \frac{\ell(\ell+1)\hbar^2}{2 r_0^2} \cdot \frac{m_K + m_{Br}}{m_K m_{Br}}$$

Lowest two rotation energy levels are $\ell = 0, 1$

so

$$\Delta E_r = \frac{\hbar^2}{2 r_0^2}\left(\frac{m_K + m_{Br}}{m_K m_{Br}}\right) = \frac{(1.05 \times 10^{-34}\ J\cdot s)^2}{2(3 \times 10^{-10} m)^2}\left(\frac{119}{80 \cdot 39}\right)\left(\frac{1}{1.7 \times 10^{-27} Kg}\right)$$

$$= 1.4 \times 10^{-24}\ J$$

$$= 8.6 \times 10^{-6}\ eV$$