Final Exam solutions

(1) a.) $1s^2\,2s^2\,2p^6\,3s^1$

b.) $$E_n = -\left(\frac{kZe^2}{\hbar}\right)^2 \frac{m_e}{2n^2}$$

Outer electron has $n=3$, inner electrons screen to give $Z \approx 1$

$$E_{ion} = \frac{k^2 e^4 m_e}{\hbar^2\, 2 \cdot 9} = \frac{13.6\ eV}{9}$$

c.) $l=0$, $s=\frac{1}{2}$ net contribution from outer electron

$J=\frac{1}{2}$

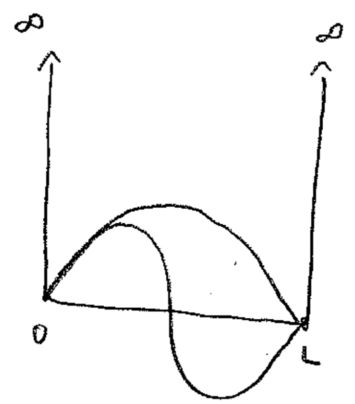
d.) The electron feels an effective magnetic field from the proton charge when $l>0$. The electron magnetic moment in this field gives energy level splitting

(2.)

a.) ground state $\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$

1st excited state $\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$

b.)



c.) fermion can't be in the same state, must have anti-symmetric wave functions

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)\right]$$

d.) $$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right]\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\sqrt{2}}\left(\left(-\frac{\pi^2}{L^2}\right)\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\left(-\frac{4\pi^2}{L^2}\right)\psi_2(x_1)\right)\right.$$

$$\left. + \frac{1}{\sqrt{2}}\left(\psi_1(x_1)\left(-\frac{4\pi^2}{L^2}\right)\psi_2(x_2) - \left(-\frac{\pi^2}{L^2}\right)\psi_1(x_2)\psi_2(x_1)\right)\right] = E\psi$$

$$\frac{\hbar^2}{2m}\left[\frac{\pi^2}{L^2}\psi + \frac{4\pi^2}{L^2}\psi\right] = E\psi \qquad E = \frac{5\pi^2\hbar^2}{2mL^2}$$

Bosons could both be in the ground state, then $E = \frac{\pi^2\hbar^2}{mL^2}$

(3.)

$$\frac{g_1 e^{-E_1/kT}}{g_0 e^{-E_0/kT}} = 1$$

$$E = \frac{L^2}{2I} = \frac{\ell(\ell+1)\hbar^2}{2I} \qquad I = 2m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$$

$$E = \frac{\ell(\ell+1)\hbar^2}{md^2}$$

ground state is $\ell=0$. $m_\ell=0$, $g_0=1$

first excited state is $\ell=1$. $m_\ell = -1,0,1$, $g_1=3$

$$E_0 = \frac{\hbar^2}{md^2} \qquad E_1 = \frac{2\hbar^2}{md^2}$$

$$3e^{-\hbar^2/md^2KT} = 1$$

$$\ln 3 = \frac{\hbar^2}{md^2KT}$$

$$\boxed{T = \frac{\hbar^2}{md^2 k \ln 3}}$$

Note if $g_1 = g_0$. $\ln 3 \to \ln 1 = 0$

only equal at $T = \infty$

(4.)

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E = \gamma m c^2$$

Energy in neutron decay is

$$m_n - m_p - m_e - \overset{\text{0 neglect}}{\cancel{m_{\nu_e}}} = (939.57 - 938.27 - 0.511)\ \text{MeV}/c^2$$

$$\approx 0.8\ \text{MeV}/c^2$$

neutrino mass is $\sim 2\ \text{eV}/c^2$

for neutrino $E_K \sim E = \gamma m c^2$

$$\gamma = \frac{E}{mc^2} = \frac{8\times10^5\ \text{eV}/c^2}{2\ \text{eV}/c^2} = 4\times10^5$$

$$V = c - \varepsilon$$

$$\gamma = \frac{1}{\sqrt{1-\frac{(c-\varepsilon)^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{c^2-2c\varepsilon-\varepsilon^2}{c^2}}} \approx \frac{1}{\sqrt{\frac{2\varepsilon}{c}}}$$

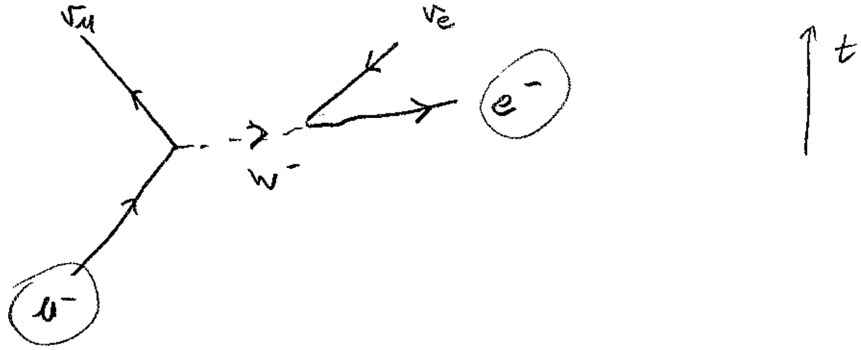
$$\varepsilon = \frac{c}{2\gamma^2}$$

$$\Delta t = \frac{\ell}{v} - \frac{\ell}{c} = \ell\left(\frac{c-v}{vc}\right) \approx \frac{\ell\varepsilon}{c^2} = \frac{\ell}{2c\gamma^2}$$

$$= \frac{(\cancel{2}\times10^5\ \text{year}\cdot\cancel{c})(3\times10^7\ \text{s/year})}{\cancel{2}\,\cancel{c}\,(1.6\times10^{11})} \approx 20\ \text{s}$$

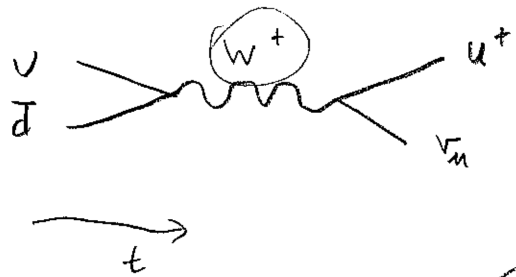
5.

a.)



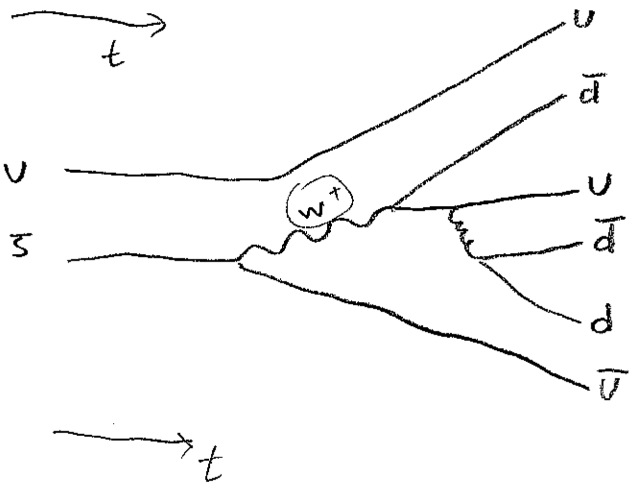
ν_μ
$\bar{\nu}_e$
e^-
W^-
μ^-
t

b.)



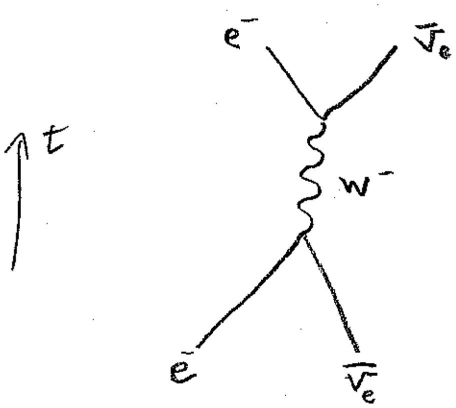
u
$\bar{d}$
W^+
μ^+
ν_μ
t

c.)



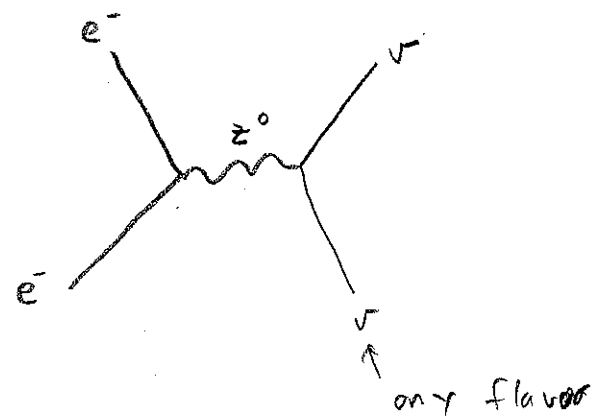
u
s
W^+
u
$\bar{d}$
u
$\bar{d}$
d
$\bar{u}$
t

d.)



t
e^-
$\bar{\nu}_e$
W^-
e^-
$\bar{\nu}_e$

or



e^-
ν
Z^0
e^-
ν
any flavor