330 Hw #10 Solutions

(1.)

$j = \sigma E$

$$R = \frac{1}{\sigma}\frac{\ell}{A} = \frac{\rho \ell}{A}$$

$$j = \frac{I}{A}$$

$$V = E\ell$$

$$\frac{I}{\cancel{A}} = \left(\frac{\cancel{\ell}}{R\cancel{A}}\right)\left(\frac{V}{\cancel{\ell}}\right)$$

$$V = IR$$

(2.)

$$E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

$$= \frac{h^2}{2m}\left(\frac{3}{8\pi}\right)^{2/3} n^{2/3}$$

$n = \frac{N}{V}$ number density

$n = \frac{1}{r_0^3}$

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\right)^{2/3}\frac{1}{r_0^2} = \frac{(hc)^2}{2mc^2}\left(\frac{3}{8\pi}\right)^{2/3}\frac{1}{r_0^2}$$

$r_0 = 0.5 \text{ nm}$

$$E_F = \frac{(1240 \text{ eV}\cdot\text{nm})^2}{2 \times 0.511 \times 10^6 \text{ eV}}\left(\frac{3}{8\pi}\right)^{2/3}\frac{1}{(0.5 \text{ nm})^2}$$

$$= 1.5 \text{ eV}$$

(3.)

Si has 4 valence electrons

a.) Al has 3 valence electrons

- p-type

b.) P has 5 valence electrons

n-type.

(4.)

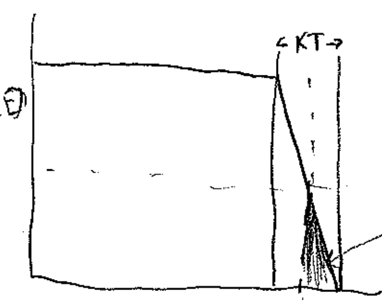
E_F copper = 7.06 eV

T_F = 8.19×10^4 K

for an exact answer

$$\frac{N_{E>E_F}}{N} = \int_{E_F}^{\infty} n(E)\, dE$$

A very crude estimate: take $n(E) \propto f_{FD}(E)$



$$\frac{N_{E>E_F}}{N} \sim \frac{\frac{1}{4}kT}{E_F} \sim \frac{kT}{E_F} = \frac{T}{T_F}$$

a.) T= 300 K $\frac{T}{T_F} = 3.7 \times 10^{-3}$

b) T= 100 K $\frac{T}{T_F} = 12.2 \times 10^{-3}$

(5.)

$n = 10^{16}\ cm^{-3} = 10^{24}\ m^{-3}$

$n_{copper} = 9 \times 10^{28}\ m^{-3}$

$\lambda_{copper} \approx 0.38\ nm$

$\rho = 5 \times 10^{-3}\ \Omega \cdot m$

$\rho_{copper} = 1.7 \times 10^{-8}\ \Omega \cdot m$

$m^* = 0.2\ m_e$

$T = 300\ K$

$$\rho = \frac{m_e \langle v \rangle}{n e^2 \lambda} \longrightarrow \frac{m^* u_F}{n e^2 \lambda}$$

calculated with m_e

$$u_F = \left(\frac{2E_F}{m_e}\right)^{1/2}$$

$$E_F = \frac{h^2}{2m_e}\left(\frac{3}{8\pi}\right)^{2/3} n^{2/3}$$

$$\lambda = \frac{m^* u_F}{n e^2 \rho} = \frac{m^*}{n e^2 \rho}\left[\frac{h^2}{m_e^2}\left(\frac{3}{8\pi}\right)^{2/3} n^{2/3}\right]^{1/2}$$

$$= \left(\frac{m^*}{m_e}\right)\left(\frac{h}{e^2 \rho}\right)\left(\frac{3}{8\pi}\right)^{1/3} \frac{1}{n^{2/3}}$$

$\lambda = .050\ nm$

much smaller than copper,

resistivity increase doesn't just come from density difference.

(6)

Hall Effect:
A voltage is generated across a conductor when current flows in a direction perpendicular to an applied electric field. This is due to the force on the charge carriers, $q\mathbf{v} \times \mathbf{B}$. The sign of the voltage indicates the type of majority charge carriers (electrons or holes).

Quantum Hall Effect:
In thin conductors, the observed Hall voltage is quantized to integer multiples of a particular voltage value. The quantization is due to quantization of the size of electron orbitals, which are the trajectories that electrons would take in a crossed electric (driving the current) and magnetic field (applied).