HW # 12 Solutions

(1.) The odd nucleon gives the net contribution to the nuclear spin.

Protons and neutrons fill up shells independently.

$^{29}_{14}Si$ $Z = 14$, $N = 15$ odd nucleon in $2s_{1/2}$, $I = \frac{1}{2}$ ↑ nuclear equivalent of J

$^{37}_{17}Cl$ $Z = 17$, $N = 20$ " $1d_{3/2}$, $I = 3/2$

$^{71}_{31}Ga$ $Z = 31$, $N = 40$ " $2p_{3/2}$, $I = 3/2$

$^{59}_{27}Co$ $Z = 27$, $N = 32$ $1f_{7/2}$ $I = 7/2$

$^{73}_{32}Ge$ $Z = 32$, $N = 41$ $1g_{9/2}$ $I = 9/2$

$^{33}_{16}S$ $Z = 16$, $N = 17$ $1d_{3/2}$ $I = 3/2$

$^{87}_{38}Sr$ $Z = 38$, $N = 49$ $1g_{9/2}$ $I = 9/2$

(2.) calculate Q values

element	Mass
^1H	1.007825 u
^2H	2.014102 u
^3He	3.016029 u
^4He	4.002602 u
e^+	.51100 MeV/c^2
ν_e	~0 eV
γ	0 eV
u	931.49401 MeV/c^2

1.) $^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e$

$$Q_1 = 2m_{^1H}c^2 - m_{^2H}c^2 - 2m_e c^2$$

↑ one electron lost since Z=1 in ^2H

2.) $^2H + {}^1H \rightarrow {}^3He + \gamma$

$$Q_2 = m_{^2H}c^2 + m_{^1H}c^2 - m_{^3He}c^2$$

3.) $^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H + \gamma$

$$Q_3 = 2m_{^3He}c^2 - m_{^4He}c^2 - 2m_{^1H}c^2$$

$Q_1 + Q_2 + Q_3 =$

$$2m_{^1H}c^2 - m_{^2H}c^2 - 2m_e c^2 + m_{^2H}c^2 + m_{^1H}c^2 - m_{^3He}c^2$$
$$+ 2m_{^3He}c^2 - m_{^4He}c^2 - 2m_{^1H}c^2$$

$$= m_{^1H}c^2 + m_{^3He}c^2 - m_{^4He}c^2 - 2m_e c^2$$

$$= (1.007825 + 3.016029 - 4.002602)\,931.49401\text{ MeV} - 2(0.51100\text{ MeV})$$

$$= 18.8\text{ MeV}$$

(3.)

In a living organism

$$^{14}C / ^{12}C = 1.35 \times 10^{-12}$$

$\tau_{1/2}$ of ^{14}C is 5730 years

The percentage of ^{12}C is so low that we can take the orginal mass of Carbon to be 15g.

Calculate initial amount of ^{12}C

$$N_0 = 15\,g \times \frac{1\ \text{Mole}}{12.0107\ g} \times \frac{6.022 \times 10^{23}}{\text{Mole}} \times 1.35 \times 10^{-12}$$

$$N_0 = 1.02 \times 10^{12}$$

10,000 years is $n = \left(\frac{10\,000}{5730}\right)$ half lives

Amount of ^{12}C remaining is $N_0 \left(\frac{1}{2}\right)^n$

$$R = \lambda N = \frac{0.693}{t_{1/2}} N_0 \left(\frac{1}{2}\right)^n$$

$$= \left(\frac{0.693}{5730\ y}\right)\left(1.02 \times 10^{12}\right)\left(\frac{1}{2}\right)^{\left(\frac{10000}{5730 y}\right)} = 3.68 \times 10^{7} \text{ per year}$$

$$= 1.17\ s^{-1}$$

Note: we could Also work from

$$N = N_0 e^{-\lambda t}$$ instead of using ½ lives

(4.)

a.) $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$

Lepton number, Baryon number, charge is conserved

Involves both leptons and hadrons, so must involve weak interaction.

b.) $p + e^- + \nu_e \rightarrow e^- + \pi^+ + p$

Lepton number, charge not conserved. Not Allowed!

c.) $\Lambda^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$

Baryon number not conserved. Not Allowed.

d.) $p + \bar{\nu}_\mu \rightarrow \mu^+ + n$

Not Allowed. Lepton number not conserved

5.

	Q	B	I_3	S	Hadron
a.) $u\bar{d}$	+1	0	+1	0	π^+
b.) $d\bar{u}$	-1	0	-1	0	π^-
c.) $u\bar{s}$	+1	0	$+\frac{1}{2}$	+1	K^+
d.) $s\bar{s}$	0	0	0	0	ϕ ← this is a "strange" one.
e.) $d\bar{s}$	0	0	$-\frac{1}{2}$	+1	K^0

Note: the convention is that (strange) ness
(Top) ness
(up) ness
⋮
has the same sign as the charge