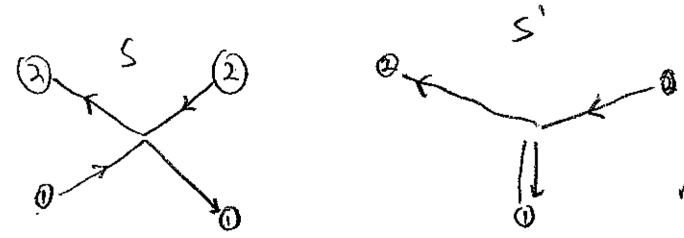
HW # 1.1 $\boxed{\text{15 points}}$



$m_1 = m_2 = m$

In S`, the particle velocities are

$\vec{u}_1$	u_2	
$\left(0, \frac{b}{\gamma(1-\beta^2)}\right)$	$\left(-\frac{2a}{(1+\beta^2)}, \frac{-b}{\gamma(1+\beta^2)}\right)$	before
$\left(0, \frac{-b}{\gamma(1-\beta^2)}\right)$	$\left(\frac{-2a}{(1+\beta^2)}, \frac{b}{\gamma(1+\beta^2)}\right)$	after

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$ v - Ref frame velocity

$$\beta \equiv \frac{a}{c}$$

Relativistic momentum

$$\vec{p} = \gamma_u m \vec{u} \qquad \gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$\vec{u}$ - particle velocity

$$u^2 = u_x^2 + u_y^2$$

In the S` frame

particle 1 $$u_1^2 = \frac{b^2}{\gamma^2(1-\beta^2)^2}$$ - same for before and after

particle 2 $$u_2^2 = \frac{4a^2}{(1+\beta^2)^2} + \frac{b^2}{\gamma^2(1+\beta^2)}$$ - same - before and after

HW# 11. cont.

first, x-components of momentum

$$\frac{p_{1x} \quad + p_{2x}}{}$$

$$0 + \left(\frac{1}{\sqrt{1-\frac{u_2^2}{c^2}}}\right) m \left(\frac{-2a}{(1+\beta^2)}\right) \quad \text{before}$$

$$0 + \left(\frac{1}{\sqrt{1-\frac{u_2^2}{c^2}}}\right) m \left(\frac{-2a}{(1+\beta^2)}\right) \quad \text{after}$$

equal, so $\vec{p} = \gamma m \vec{u}$ is conserved in x-direction

y - components

$$\frac{p_{1y} \quad + p_{2y}}{}$$

before

$$\left(\frac{1}{\sqrt{1-\frac{b^2}{c^2\gamma^2(1-\beta^2)^2}}}\right) m \left(\frac{b}{\gamma(1-\beta^2)}\right) + \left(\frac{1}{\sqrt{1-\frac{1}{c^2}\left(\frac{4a^2}{(1+\beta^2)^2}+\frac{b^2}{\gamma^2(1+\beta^2)^2}\right)}}\right) m \left(\frac{-b}{\gamma(1+\beta^2)}\right)$$

$$\left(\frac{1}{\sqrt{1-\frac{b^2}{c^2\gamma^2(1-\beta^2)^2}}}\right) m \left(\frac{-b}{\gamma(1-\beta^2)}\right) + \left(\frac{1}{\sqrt{1-\frac{1}{c^2}\left(\frac{4a^2}{(1+\beta^2)^2}+\frac{b^2}{\gamma^2(1+\beta^2)^2}\right)}}\right) m \left(\frac{b}{\gamma(1+\beta^2)}\right)$$

after

show before and after terms are equal.

HW # 1.1 cont.

note that $\gamma^2 = \frac{1}{1-\beta^2}$, $\beta \equiv \frac{a}{c}$ since $v = a$

$$= \frac{1}{1-\frac{a^2}{c^2}}$$

In our example $p_{1y} + p_{2y}$ should be zero,

so if we can show $p_{1y} = -p_{2y}$ before and after collision, momentum is conserved

$$p_{1y} = \frac{mb}{\gamma}\left[(1-\beta^2)\left(1-\frac{b^2}{c^2(1-\beta^2)}\right)^{\frac{1}{2}}\right]^{-1}$$

$$= \frac{mb}{\gamma}\left[(1-\beta^2)^2 - \frac{b^2(1-\beta^2)}{c^2}\right]^{-\frac{1}{2}} \longleftarrow \text{eq. 1}$$

$$p_{2y} = \frac{-mb}{\gamma}\left[(1+\beta^2)\left(1-\frac{1}{c^2}\left(\frac{4a^2}{(1+\beta^2)^2}+\frac{b^2}{\gamma^2(1+\beta^2)^2}\right)\right)^{\frac{1}{2}}\right]^{-1}$$

$$= \frac{-mb}{\gamma}\left[(1+\beta^2)^2 - 4\beta^2 - \frac{b^2}{c^2}(1-\beta^2)\right]^{-\frac{1}{2}}$$

$$= -\frac{mb}{\gamma}\left[(1-\beta^2)^2 - \frac{b^2}{c^2}(1-\beta^2)\right]^{-\frac{1}{2}} \longleftarrow \text{eq. 2}$$

$p_{1y} + p_{2y} = 0$ ← before

before and after only differ by -1

so $p_{1y} + p_{2y} = 0$ after

y-momentum is conserved

HW 1.2

$$E_K = \int F\,dx = \int_0^u \frac{dp}{dt}\,dx = \int_0^u \frac{d(\gamma m u)}{dt}$$

$$= \int_0^u u\, d(\gamma m u)$$

$$d(\gamma m v) = \gamma m\,du + um\left(-\tfrac{1}{2}\right)\left(-\tfrac{2u}{c^2}\right)\gamma^3\,du$$

$$= m\,du\,\gamma\left[1 + \frac{u^2}{c^2}\gamma^2\right]$$

$$= m\,dv\,\gamma\left[1 + \frac{u^2/c^2}{1-u^2/c^2}\right]$$

$$= m\,dv\,\gamma\left[\frac{1}{1-u^2/c^2}\right]$$

$$= \gamma^3 m\,du$$

prime means integration variable

$$\int_0^u u'\,d(\gamma m v') = \int_0^u \gamma^3 m\,u'du' = \int_0^u m\left(1-\frac{u'^2}{c^2}\right)^{-3/2} u'\,du'$$

$$x = \left(1-\frac{u'^2}{c^2}\right) \qquad dx = -\frac{2u'}{c^2}\,du'$$

$$m\int -\frac{c^2}{2}x^{-3/2}\,dx = mc^2 x^{-1/2} = mc^2\frac{1}{\sqrt{1-\frac{u'^2}{c^2}}}$$

putting in limits

$$E_K = mc^2(\gamma - 1)$$

HW # 1.3

show $\gamma_1 mc^2 + \gamma_2 mc^2$ is conserved

$$u_1^2 = \frac{b^2}{\gamma^2(1-\beta^2)}$$ - before and after are the same

$$u_2^2 = \frac{4a^2}{(1+\beta^2)^2} + \frac{b^2}{\gamma^2(1+\beta^2)}$$ - before and after are the same

$$\gamma_1 = \frac{1}{\sqrt{1-\frac{u_1^2}{c^2}}} , \quad \gamma_2 = \frac{1}{\sqrt{1-\frac{u_2^2}{c^2}}}$$

γ_1, γ_2 don't change through the collision

so $\gamma_1 mc^2 + \gamma_2 mc^2$ is conserved