Physics 330 Homework # 4

(5.5)

$$\frac{3}{2}KT = \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m}$$

$$\lambda = \frac{h}{\sqrt{3mKT}} = \frac{hc}{\sqrt{3mc^2KT}}$$

using $KT \sim \frac{1}{40}eV$ at 293 K

$m_{N_2} = 2 \cdot 14 \cdot m_u = 2 \cdot 14 \cdot 931 \text{ MeV}/c^2$

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{3\left(2 \cdot 14 \cdot 9.31 \times 10^8 \text{ eV}\right)\frac{1}{40}\text{ eV}}}$$

$= .028$ nm

(5.14)

$n\lambda = D \sin\phi$

$D = 0.215$ nm

$E_K = 54$ eV

$\sin\phi = \frac{n\lambda}{D}$

$E_K = \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m}$

$= \frac{n\,hc}{D\sqrt{2mc^2 E_K}}$

$\lambda = \frac{h}{\sqrt{2mE_K}}$

$= n \frac{1240 \text{ eV}\cdot\text{nm}}{(0.215 \text{ nm})\sqrt{2(.511\times10^6 \text{eV})(54\text{eV})}}$

$= n\,(0.78)$

$\sin\phi$ can't be greater than 1

so n can't be 2, 3, ...

(5.21)

$\Delta f = 5000$ Hz

for wave packets

$\Delta\omega \Delta t \sim 1$

$\omega = 2\pi f$

$\Delta\omega = 2\pi \Delta f$

$2\pi \Delta f \Delta t \sim 1$

$\Delta t = \frac{1}{2\pi \Delta f} = 3.2 \times 10^{-5}$ s.

(5.41)

$$E^2 = p^2c^2 + m^2c^4$$

$$E = \hbar w$$
$$p = \hbar k$$

a.) $V_p = \frac{w}{k}$

$$\hbar^2 w^2 = \hbar^2 k^2 c^2 + m^2 c^4$$

$$\frac{w}{k} = \frac{\left(k^2c^2 + \frac{m^2c^4}{\hbar^2}\right)^{1/2}}{k} = \left(c^2 + \frac{m^2c^4}{\hbar^2 k^2}\right)^{1/2}$$

$$= c\left(1 + \frac{m^2c^2}{\hbar^2 k^2}\right)^{1/2}$$

↑ always positive

$$\frac{w}{k} \geq c \quad , \quad \frac{w}{k} \rightarrow c \quad \text{as} \quad k \rightarrow \infty$$

b.) $V_g = \frac{dw}{dk}$

$$\hbar^2 w^2 = \hbar^2 k^2 c^2 + m^2 c^4$$

$$\hbar^2 (2w\,dw) = \hbar^2 (2k\,dk) c^2$$

5.41 cont.

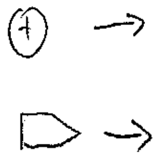
$$\frac{d\omega}{dk} = \frac{k}{\omega} c^2 \qquad \text{using part a.}$$

$$= \frac{c^2}{c\left(1 + \frac{m^2 c^2}{\hbar^2 k^2}\right)^{1/2}} \qquad p = \gamma m v$$

$$= \frac{c}{\left(1 + \frac{c^2}{v^2} \frac{1}{\gamma^2}\right)^{1/2}} = \frac{c}{\left(1 + \frac{c^2}{v^2}\left(1 - \frac{v^2}{c^2}\right)\right)^{1/2}}$$

$$= v$$

5.47



$v = 500\ m/s$

$\Delta v = \frac{.01}{100} \times 500\ m/s = 5 \times 10^{-2}\ m/s$

$\Delta p \Delta x \geq \frac{\hbar}{2}$

$m \Delta v \Delta x \geq \hbar/2$

proton.

$m = 1.7 \times 10^{-27}\ kg$

$\hbar = 1.05 \times 10^{-34}\ J \cdot s$

$$\Delta x \geq \frac{\hbar}{m \Delta v} = \frac{1.05 \times 10^{-34}\ J \cdot s}{1.7 \times 10^{-27}\ kg \cdot 5 \times 10^{-2}\ m/s} = 1.2\ \mu m$$

Bullet $m = 10^{-2}\ kg$

$$\Delta x \geq \frac{1.05 \times 10^{-34}\ J \cdot s}{10^{-2}\ kg \cdot 5 \times 10^{-2}\ m/s} = 2.1 \times 10^{-31}\ m$$