Physics 330 HW #5

1.

$$\Psi(x,t) = Ae^{kx-wt}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = E\Psi$$ time independent

$$-\frac{\hbar^2}{2m}(k^2)\Psi = E\Psi$$ ← energy would be negative, not true for $V=0$, free particle

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = i\hbar\frac{\partial}{\partial t}\Psi$$ time-dependent

$$-\frac{\hbar^2k^2}{2m} = -iw$$, w, k real, so this can't be true.

$$\Psi(x,t) = Ae^{i(kx-wt)}$$

$$-\frac{\hbar^2}{2m}(ik)^2 = i\hbar(-iw)$$ time-dependent Schrödinger

$$\frac{\hbar^2k^2}{2m} = \hbar w$$ ← O.K.

classical wave equation

$$\frac{\partial^2\Psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\Psi(x,t)}{\partial t^2}$$

$$(ik)^2 = \frac{1}{v^2}(-iw)^2$$

$$v = \frac{w}{k}$$ ← O.K.

2.

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad n=1 \quad \text{for ground state}$$

$\Delta x = .002L$

Probability distribution is $|\psi_1(x)|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)$

Approximate integral as $f(x)dx$

a.) $x = \frac{L}{2}$

$$\left(\frac{2}{L}\right)(.002L)\sin^2\left(\frac{\pi}{2}\right) = .004$$

b.) $x = \frac{2L}{3}$

$$\left(\frac{2}{L}\right)(.002L)\sin^2\left(\frac{2\pi}{3}\right) = \left(\frac{3}{4}\right)(.002)(2) = .003$$

c.) $x = L$

$$\left(\frac{2}{L}\right)(.002L)\sin^2(\pi) = 0$$

(3)

$$\lambda = 694.3 \text{ nm}$$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

$$E_2 - E_1 = \left[(2)^2 - (1)^2\right] \frac{\hbar^2 \pi^2}{2mL^2}$$

$$\Delta E = \frac{3\hbar^2\pi^2}{2mL^2} = \frac{hc}{\lambda} = \frac{\hbar 2\pi c}{\lambda}$$

$$\frac{2mL^2}{3\hbar^2\pi^2} = \frac{\lambda}{hc}$$

$$L = \left[\left(\frac{\lambda}{hc}\right)\left(\frac{3\pi^2(\hbar c)^2}{2 m_e c^2}\right)\right]^{1/2}$$

$$L = \left[\left(\frac{694.3 \text{ nm}}{1240 \text{ eV}\cdot\text{nm}}\right)\left(\frac{3\pi^2 (197 \text{ eV}\cdot\text{nm})^2}{2(.511 \times 10^6 \text{ eV})}\right)\right]^{1/2}$$

$$L = .79 \text{ nm}$$

(4)

Find $\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$

$\sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$

$\langle p\rangle = 0$

$\langle p^2\rangle = \langle 2mE\rangle = 2mE_1$

by symmetry

$\langle x\rangle = \frac{L}{2}$, $\langle x\rangle^2 = \frac{L}{2}$

$$\langle x^2\rangle = \int_0^L \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) x^2 \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{2}{L}\int_0^L x^2 \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{1}{6} L^2 \left(2 - \frac{3}{\pi^2}\right)$$

$$\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle} = \sqrt{\langle x^2\rangle - 0}$$

$$= L\left(\frac{1}{3} - \frac{1}{2\pi^2}\right)^{1/2} = 0.53\ L$$

$$\sigma_p = \sqrt{\langle p^2\rangle - 0} = \sqrt{2mE_1} = \frac{\hbar\pi}{L}$$

note

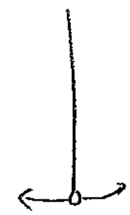
$$\sigma_x \sigma_p \simeq \frac{\pi\hbar}{2} > \frac{\hbar}{2}$$

5.

$m = 10g$

$L = 50\ cm$

$T = 1.42s$



a) $E_0 = \frac{\hbar w}{2}$

$$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\ell}}$$

pendulum

$$E_0 = \frac{\hbar}{2}\sqrt{\frac{g}{\ell}} = \left(\frac{1}{2}\right)\left(1.05\times10^{-34}\,J\cdot s\right)\sqrt{\frac{9.8\,m/s^2}{0.5\,m}}$$

$$= 2.3\times10^{-34}\ J$$

b) $E = (n+\frac{1}{2})\hbar w = mgh$ ← total energy is only potential energy at top.

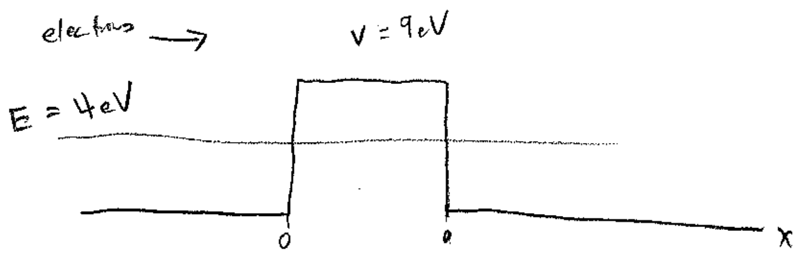
$$n = \frac{mgh}{\hbar w} - \frac{1}{2}$$

$$n = \frac{(.01\,Kg)(9.8\,m/s^2)(10^{-4}\,m)}{(1.05\times10^{-34}\,J\cdot s)\sqrt{\frac{9.8\,m/s^2}{0.5\,m}}} - \frac{1}{2} = 2.1\times10^{28}$$

c) Same as before, $f = \frac{1}{T} = 0.7\ Hz$

(6.)

This a tunneling problem



a = 0.6 nm

Is $\alpha a \gg 1$? If so, use approximation

$$\alpha = \frac{\sqrt{2m(V-E)}}{\hbar} =$$

$V - E = 5eV$

$$\alpha a = \left(\frac{2 m_e c^2 a^2 (V-E)}{\hbar^2 c^2} \right)^{1/2}$$

$$= \left(\frac{2 (.511 \times 10^6 \, eV)(0.6 \, nm)^2 (5 \, eV)}{(197 \, eV \cdot nm)^2} \right)^{1/2}$$

a.) $\alpha a = 6.9$ - ok. using approximation

$$T \approx 16 \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2\alpha a} = 16 \left(\frac{4eV}{9eV}\right)\left(1 - \frac{4eV}{9eV}\right) e^{-2(6.9)} = 4.1 \times 10^{-6}$$

(6.)

I plotted

$$T \simeq 16 \frac{E}{V}\left(1 - \frac{E}{V_0}\right) e^{-2\alpha a} \quad \text{vs} \quad E$$

and found

$$T = 8.2 \times 10^{-6}$$

at

$$E \sim 4.45 \text{ eV}$$

so $V = 4.45$ V