Physics 330 HW #6

1.

$$E_{n_1 n_2 n_3} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$$

$L_2 = 2L_1$
$L_3 = 3L_1$

$$E_{n_1 n_2 n_3} = \frac{\hbar^2 \pi^2}{2mL_1^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{9}\right)$$

n_1	n_2	n_3	$E \Big/ \left(\frac{\hbar^2\pi^2}{2mL_1^2}\right)$
1	1	1	1.36
1	1	2	1.69
1	2	1	2.11
1	1	3	2.25
1	2	2	2.44
1	2	3	3.00
1	1	4	3.028
1	3	1	3.36
1	3	2	3.69
1	2	4	3.78

could express in fractions

see Matlab code →

```
clear n1array n2array n3array Earray
count=1;
for n1=1:10
    for n2=1:10
        for n3=1:10
            n1array(count)=n1;
            n2array(count)=n2;
            n3array(count)=n3;
            Earray(count)=n1^2+n2^2/4+n3^2/9;
            count=count+1;
        end
    end
end

Earray

%re-order array using sort

[SortedEarray listorder]=sort(Earray);

SortedArrays=[n1array(listorder)' n2array(listorder)'  n3array(listorder)' SortedEarray'];

SortedArrays(1:10,:)


ans =

    1.0000    1.0000    1.0000    1.3611
    1.0000    1.0000    2.0000    1.6944
    1.0000    2.0000    1.0000    2.1111
    1.0000    1.0000    3.0000    2.2500
    1.0000    2.0000    2.0000    2.4444
    1.0000    2.0000    3.0000    3.0000
    1.0000    1.0000    4.0000    3.0278
    1.0000    3.0000    1.0000    3.3611
    1.0000    3.0000    2.0000    3.6944
    1.0000    2.0000    4.0000    3.7778
```

(2.) $n=3$

a.) $\ell = 0, 1, 2$

b.) $\left.\begin{array}{ll} \ell=0, & m=0 \\ \ell=1, & m=-1,0,1 \\ \ell=2, & m=-2,-1,0,1,2 \end{array}\right\}$ 9 states

c.) $2 \times 9 = 18$ possible states with $n=3$

(3.)

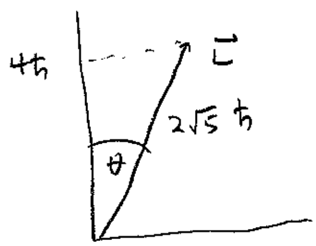
a.) $\ell = 4$

$L = \sqrt{\ell(\ell+1)}\,\hbar = 2\sqrt{5}\,\hbar$

$\vec{L}$ is quantized along z in units of $\hbar$

$m_\ell = 4$

$L_z = 4\hbar$



$\cos\theta = \frac{4}{2\sqrt{5}}$

$\theta = 0.46 \text{ rad} = 27^\circ$

b.) $m_\ell = 2$, $L_z = 2\hbar$

$L = \sqrt{6}\,\hbar$

$\cos\theta = \frac{2}{\sqrt{6}} = 0.62 \text{ rad} = 35^\circ$

4. $n = 3, \ l = 2, \ m_l = -1$

$$\psi(r,\theta,\phi) = Y_{l,m}(\theta,\phi) \ R_{n,l}(r)$$

$$= \sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta \, e^{-i\phi} \frac{4}{81\sqrt{30a_0^3}} \frac{r^2}{a_0^2} e^{-r/3a_0}$$

$$= \left(\frac{1}{\sqrt{\pi} \ a_0^{5/2} \ 81}\right) \sin\theta\cos\theta \, e^{-i\phi} \ r^2 e^{-r/3a_0}$$

check for normalization

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} r^2 dr \sin\theta \, d\theta \, d\phi \ |\psi(r,\theta,\phi)|^2$$ should equal 1

$$\frac{1}{\pi a_0^5 \, 81^2} \underbrace{\int_0^{2\pi} e^{-i\phi} e^{i\phi} \, d\phi}_{2\pi} \ \underbrace{\int_0^{\pi} \sin^3\theta \cos^2(\theta) \, d\theta}_{\frac{4}{15}} \ \underbrace{\int_0^{\infty} r^6 e^{-2r/3a_0} \, dr}_{\frac{3}{8} a^5 \cdot 32805}$$

$$= 1$$

5.

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{3}{4}}\,\hbar$

$r = 10^{-15}\,m$

$L = I\omega$

$\omega = 2\pi f$

$v = \omega r$

$L = I\frac{v}{r}$

$I_{sphere} = \frac{2mr^2}{5}$

$v = \frac{rL}{I}$

$$v = \frac{(10^{-15}\,m)\left(\sqrt{\frac{3}{4}}\;1.05\times10^{-34}\,J\cdot s\right)}{\frac{2}{5}\cdot(9.1\times10^{-31}\,kg)(10^{-15}\,m)^2}$$

$v = 2.5\times10^{11}\,m/s$

$v > c$!

6.

a.) $j = 3/2$

$$F = \mu_z \frac{dB}{dz}$$

$$\mu_z \propto J_z$$

$$J_z = \hbar m_j$$

$$m_j = -3/2, -1/2, 1/2, 3/2$$

four possible lines

b.) $j = l+s$ or $j = |l-s|$

if $\vec{S} = 0$

$$\vec{J} = \vec{L}$$

$$m_j = -1, 0, 1$$

three lines