Physics 330 HW #7

(1.) H-atom in 3D state

a.) possible values of j.

$\ell = 2$

$s = \frac{1}{2}$

$j = \frac{3}{2}, \frac{5}{2}$

b.) $J = \sqrt{j(j+1)}\,\hbar$

$\frac{3}{2}: \sqrt{\frac{3}{2}\left(\frac{5}{2}\right)}\,\hbar = \frac{\sqrt{15}}{2}\hbar$

$\frac{5}{2}: \sqrt{\frac{5}{2}\left(\frac{7}{2}\right)}\,\hbar = \frac{\sqrt{35}}{2}\hbar$

c.) $J_z = m_z \hbar$, $m_z = -j, -j+1, \cdots j$

$J_z = -\frac{5}{2}\hbar, -\frac{3}{2}\hbar, -\frac{1}{2}\hbar, \frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar$

(2.) $\ell = 1, \; s = \frac{1}{2}$

a.) possible values of ℓ corresponding to $\vec{L} = \vec{L}_1 + \vec{L}_2$

$\ell = 0, 1, 2$

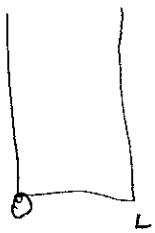
b.) $s = 0, 1$

c.) $\vec{J} = \vec{L} + \vec{S}$, $j = 0, 1, 2, 3$

d.) $j_1, j_2 = \frac{1}{2}, \frac{3}{2}$

e.) $0, 1, 2, 3$

3.



two neutrons

$L = 2 \times 10^{-15}$ m

Can't both be in $n=1$ state

$$\psi_1(x_1) = \sqrt{\frac{2}{L}} \sin \frac{\pi x_1}{L}$$

$$\psi_2(x_2) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x_2}{L}$$

make anti-symmetric wave function

$$\psi(x_1, x_2) = \sqrt{2}\left[\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)\right]$$

Schrödinger eq.

$$-\frac{\hbar^2}{2m}\left(\nabla_1^2 + \nabla_2^2\right)\psi(x_1, x_2) = E\,\psi(x_1, x_2)$$

using $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$

$$\frac{1}{\sqrt{2}}\left[E_1\psi_1(x_1)\psi_2(x_2) - E_2\psi_2(x_1)\psi_1(x_2)\right]$$

$$+ \frac{1}{\sqrt{2}}\left[E_2\psi_1(x_1)\psi_2(x_2) - E_1\psi_2(x_1)\psi_1(x_2)\right] = E_{tot}\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)\right]$$

$$= \left(E_1 + E_2\right)\psi(x_1, x_2) = E_{tot}\,\psi(x_1, x_2)$$

$$E_{tot} = E_1 + E_2 = \frac{5\pi^2\hbar^2}{2mL^2} = \frac{5\pi^2\left(1.055 \times 10^{-34}\ J\cdot s\right)^2}{2\cdot\left(1.67 \times 10^{-27}\ kg\right)\left(2.0 \times 10^{-15}\ m\right)^2}$$

as expected

$$E_{tot} = 4.1 \times 10^{-11}\ J = 25\ MeV$$

(4.)

a.) $Z = 6$

$1s^2\ 2s^2\ 2p^2$

b.) $Z = 8$

$1s^2\ 2s^2\ 2p^4$

c.) $Z = 18$

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

(5.)

$4S_{1/2} \rightarrow 3S_{1/2}$ forbidden $\Delta l \neq \pm 1$

$4S_{1/2} \rightarrow 3P_{3/2}$ OK

$4P_{3/2} \rightarrow 3S_{1/2}$ OK

$4D_{5/2} \rightarrow 3P_{1/2}$ forbidden $\Delta J \neq \pm 1, 0$, also $\Delta m_s \neq 0$

$4D_{3/2} \rightarrow 3P_{1/2}$ OK

$4D_{3/2} \rightarrow 3S_{1/2}$ forbidden $\Delta l \neq \pm 1$

$5D_{5/2} \rightarrow 4S_{1/2}$ forbidden $\Delta J \neq \pm 1, 0$ also $\Delta m_s \neq 0$

$5P_{1/2} \rightarrow 3S_{1/2}$ OK.

(6.)

a.) $E = \frac{hc}{\lambda}$

$4p_{3/2}$ $E = \frac{1240\ eV \cdot nm}{766.41\ nm} = 1.618\ eV$

$4p_{1/2}$ $E = \frac{1240\ eV \cdot nm}{769.90} = 1.611\ eV$

b) $\Delta E = 7.334\ meV$

c.) $\mu_z = -m_s g_s \mu_B$

$\Delta E = 2U$

$U = -\mu_z B_z$

$\Delta E = 2 B_z m_s g_s \mu_B$

$\approx 2 B_z \mu_B$

$B \approx \frac{\Delta E}{2\mu_B} = \frac{(7.334 \times 10^{-3}\ eV)}{2(5.79 \times 10^{-5}\ eV/T)}$

$B \approx 63.3\ T$