330 HW #8

(1.) a.) $\frac{1}{2}KT$ on average for each molecule, each degree of freedom.

Translation only

$$E_K = \frac{3}{2} KT \cdot N_A = \frac{3}{2} RT$$

$$= \frac{3}{2} \left(8.31 \frac{J}{mol\,K} \right) (273\ K) \cdot 1\ mole$$

$$= 3.4 \times 10^3\ J.$$

b.) the same for He since mass or molecular structure doesn't change average translational energy.

2.

$$n(v)\,dv = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv$$

$$\langle v^2 \rangle = \int_0^\infty v^2\, n(v)\,dv$$

$$= 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^4 e^{-mv^2/2kT}\,dv$$

$$x = \left(\frac{m}{2kT}\right)^{1/2} v$$

$$dx = \left(\frac{m}{2kT}\right)^{1/2} dv$$

$$= 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^{5/2} \underbrace{\int_0^\infty x^4 e^{-x^2}\,dx}_{\frac{3}{8}\sqrt{\pi}}$$

$$= (4N)\left(\frac{3}{8}\right)\left(\frac{2kT}{m}\right) = N\frac{3kT}{m} \rightarrow \frac{3kT}{n}$$ average per molecule

Check:

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$\langle v^2 \rangle = \frac{3kT}{m}$$

(3.)

$$\frac{N_2}{N_1} = \frac{g_2 \, e^{-E_2/kT}}{g_1 \, e^{-E_1/kT}} = \frac{g_2}{g_1} \, e^{-(E_2 - E_1)/kT}$$

$$= 3 \, e^{-(4 \times 10^{-3} \, eV) / \frac{1}{40} \, eV}$$

$$= 2.6$$

(4.)

Energy per mole

$$U = N_A \left(\tfrac{1}{2}+\tfrac{1}{2}\right) kT = RT$$

$$c_v = \left(\frac{\partial U}{\partial T}\right)_V = R$$

$$c_p = c_v + nR$$

Defined c_v as molar heat capacity so $n = 1$ (1 moles

$$c_p = 2R$$

$$\gamma = \frac{c_p}{c_v} = 2$$

5.

$$\frac{N_0}{N} \approx 1 - \left(\frac{T}{T_C}\right)^{3/2}$$

a.) $T = \frac{3}{4} T_C$

$$\frac{N_0}{N} \approx 1 - \left(\frac{3}{4}\right)^{3/2} = 0.35$$

b.) $1 - \left(\frac{1}{2}\right)^{3/2} = 0.65$

c.) $1 - \left(\frac{1}{4}\right)^{3/2} = 0.88$

d.) $1 - \left(\frac{1}{8}\right)^{3/2} = 0.96$

6.

$$C_V = 3N_A k \left(\frac{hf}{kT}\right)^2 \frac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$$

a.) $T \to \infty$

exponent argument is small

$$e^{hf/kT} \approx 1 + \frac{hf}{kT}$$

small

$$C_V \approx 3N_A k\left(\frac{hf}{kT}\right)^2 \frac{1 + hf/kT}{\left(hf/kT\right)^2}$$

$$= 3N_A k = 3R$$

b.) $T \to 0$, show $C_V \to 0$

same as showing

$$x^2 \frac{e^x}{(e^x-1)^2} \to 0 \quad \text{as} \quad x \to \infty$$

this term $\to 1$ as $x \to \infty$

$$= \frac{x^2}{e^x-1} \boxed{\frac{e^x}{(e^x-1)}}$$

exponent gets large faster than polynomial, or use L'hopital's rule

so $C_V \to 0$