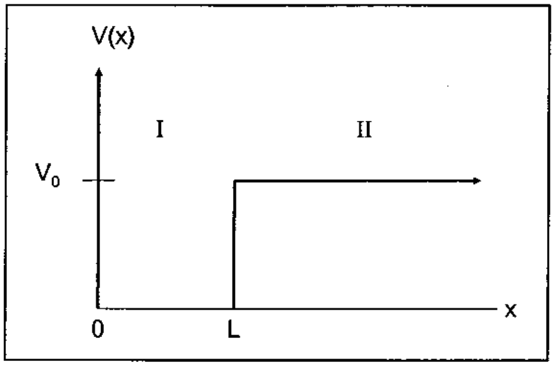
Physics 330 Midterm Exam 1
Feb. 9, 2007
100 points total

1) A particle is confined to the potential shown below, which has aspects of both the 1D particle in a box, and the finite square well.

   a) (10 pts) Write the time-independent Schrödinger equation for region I.
   b) (10 pts) Write the time-independent Schrödinger equation for region II.
   c) (10 pts) What are the boundary conditions that $\Psi_I(x)$ and $\Psi_{II}(x)$ must satisfy?
   d) (10 pts) Draw the potential and the ground state wave function.
   e) (10 pts) What is the ground state wave function in region II? Express in terms of V_0, E_1, x, a normalization constant, and any other necessary physical constants.
   f) (10 pts) What is the probability of finding the particle at $x>L$? Setup the integral, but do not solve.



2) (15 pts) I have an electron trapped in a 1D box of size L. At time t=0, I open the box (The potential vanishes, and the particle is free). Use the uncertainty principle to estimate the how large of region must be searched in order to have a good chance of finding the particle at time t.

3) (10 pts) Two types of nuclei undergo alpha decay at rates that differ by more than two orders of magnitude. An alpha particle ejected from the first is found to have a kinetic energy of 4 MeV, and that from the second, 5 MeV. Which nuclei has the longer lifetime? Why? Why is there such a difference in lifetimes?

4) In a particular reference frame, S, a proton and electron are moving directly towards each other. The proton is moving with speed c/10 (a relativistic speed). The total momentum of the two particle system is zero. Use $m_p=2000*m_e$. First find expressions in terms of m_e, m_p, v_p, and γ_p, then simplify if you have time.

   a) (5 pts) How fast is the electron moving in terms of c?
   b) (5 pts) What is the total energy of the system? Express in terms m_e and c.
   c) (5 pts) What is the wavelength of the electron? Express in m_e, c, and any other necessary physical constants.

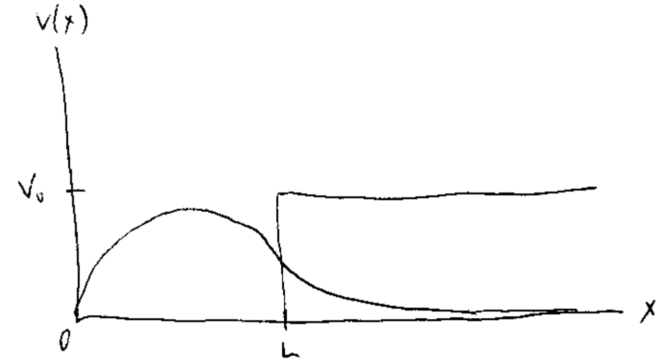
Exam 1

1.) a.) $$-\frac{\hbar^2}{2m}\frac{d^2\psi_I(x)}{dx^2} = E\,\psi_I(x)$$

b.) $$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}(x)}{dx^2} + V_0\,\psi_{II}(x) = E\,\psi_{II}(x)$$

c.) $$\psi_I(0) = 0$$
$$\psi_{II}(L) = \psi_I(x)$$
$$\frac{d\,\psi_I(x)}{dx} = \frac{d\,\psi_{II}(x)}{dx} \quad \text{at} \quad x = L$$

d.)



e.) $$\psi_{II}(x) = C e^{-\beta x} \qquad \beta \equiv \frac{\sqrt{2m(V_0-E)}}{\hbar}$$
$$\psi_{II}(x) = C\, e^{-\frac{x}{\hbar}\sqrt{2m(V_0-E)}}$$

f.) $$P = C^2 \int_L^\infty e^{-\frac{2x}{\hbar}\sqrt{2m(V_0-E)}}$$

2.) At $t=0$ $\Delta x_0 = L$

The uncertainty in position at time t is from the uncertainty of the momentum at $t=0$

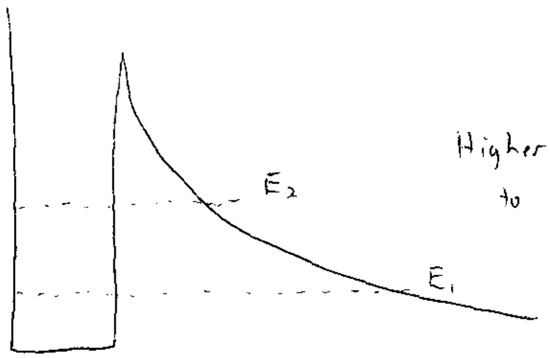
$$\Delta x_t = \Delta V t = \frac{\Delta p_0}{m} t$$

$$\Delta x_0 \Delta p_0 \approx \hbar$$

$$\Delta x_t \approx \frac{\hbar t}{\Delta x_0 m} = \frac{\hbar t}{L m}$$

3.) The nuclei that eject 4 MeV alpha particles have longer $t_{1/2}$ since higher energies need to tunnel a smaller distance before they are free. The lifetimes are so different because the energy appears in the exponent:

$$P = A^2 e^{-\frac{4 Z k e^2}{\hbar E_k} \sqrt{2m(V_0 - E_k)}}$$



Higher energy particles need to tunnel through a smaller barrier.

4.)

b.) $$E_{total} = \gamma_p m_p c^2 + \gamma_e m_e c^2$$

$$\gamma_e = \left[1 - \frac{V_e^2}{c^2}\right]^{-1/2}$$

$$\alpha \equiv \frac{\gamma_p m_p V_p}{m_e} = \frac{200c}{\sqrt{99}}$$

$$V_e = \left[\frac{1}{\frac{1}{c^2} + \frac{1}{\alpha^2}}\right]^{1/2}$$

$$\gamma_p = \frac{10}{\sqrt{99}}$$

$$\frac{V_e^2}{c^2} = \frac{1}{1 + \frac{c^2}{\alpha^2}}$$

$$\gamma_e = \left[1 - \frac{1}{1 + \frac{c^2}{\alpha^2}}\right]^{-1/2} = \left[\frac{c^2/\alpha^2}{1 + c^2/\alpha^2}\right]^{-1/2} = \left[\frac{\alpha^2}{c^2} + 1\right]^{1/2}$$

$$E_{total} = \left(\frac{10}{\sqrt{99}}\right)\left(2000\, m_e\right)c^2 + \left[\frac{4 \times 10^4}{99} + 1\right]^{1/2} m_e c^2$$

c.) $$\lambda = \frac{h}{p_e} = \frac{h}{\gamma_e m_e V_e} = \frac{h}{\gamma_p m_p V_p} = \frac{h}{\left(\frac{10}{\sqrt{99}}\right)\left(2000\, m_e\right)\left(\frac{c}{10}\right)}$$

$$= \frac{3h\sqrt{11}}{2000\, m_e c}$$

4.)

a.) $\gamma_p m_p V_p = \gamma_e m_e V_e$

$$\gamma_e V_e = \frac{\gamma_p m_p V_p}{m_e} \equiv \alpha$$

↑ simplify expressions for now ..

$$\gamma_e = \frac{1}{\sqrt{1 - \frac{V_e^2}{c^2}}}$$

$$\gamma_e V_e = \frac{V_e}{\sqrt{1 - \frac{V_e^2}{c^2}}} = \frac{1}{\sqrt{\frac{1}{V_e^2} - \frac{1}{c^2}}} = \alpha$$

$$\frac{1}{V_e^2} - \frac{1}{c^2} = \frac{1}{\alpha^2}$$

$$V_e = \left[\frac{1}{\frac{1}{c^2} + \frac{1}{\alpha^2}}\right]^{1/2} = \left[\frac{1}{\frac{1}{c^2} + \frac{m_e^2}{\gamma_p^2 m_p^2 V_p^2}}\right]^{1/2}$$

simplify

$$\frac{1}{\alpha} = \frac{m_e}{\gamma_p m_p V_p} = \frac{\sqrt{1 - \frac{V_p^2}{c^2}}}{2000\, V_p} = \frac{\sqrt{1 - \left(\frac{1}{10}\right)^2}}{2000\, c/10} = \frac{\sqrt{99}}{200c}$$

$$\frac{1}{\alpha^2} = \frac{99}{4\times10^4 c^2}$$

$$V_e = \left[\frac{1}{\frac{1}{c^2} + \frac{99}{4\times10^4 c^2}}\right]^{1/2} = c\left[1 + \frac{99}{4\times10^4}\right]^{-1/2} \approx c\left[1 - \frac{99}{8\times10^4}\right]$$

$$\approx c\left[1 - \frac{1}{1000}\right] = 0.999c$$