Relativity

Postulates

1. The laws of physics are identical in all inertial frames of reference

2. The speed of light (all EM radiation) in vacuum is constant, independant of motion of the source

Lorentz Transformations

$$t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time dilation

$$\Delta t' = \gamma \Delta t$$

Length contraction

$$L = \frac{L_0}{\gamma}$$

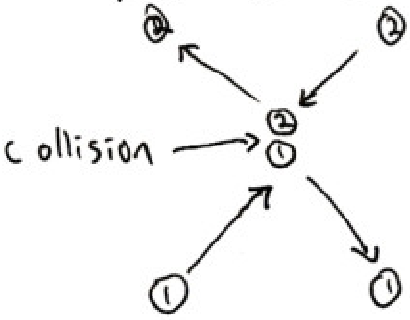
Relativistic Momentum

classical definition $\vec{p} \equiv m\vec{v}$

momentum is conserved in all inertial frames

what about when making relativistic transformations between reference frames?

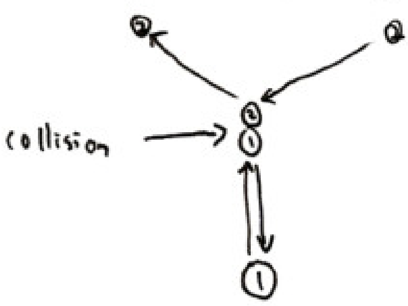
Analyise an elastic collision between equal mass balls



inertial frame S

$\vec{v}_1$	$\vec{v}_2$	$\vec{p}_1 + \vec{p}_2$	
(a,b)	$(-a,-b)$	$(0,0)$	before
$(a,-b)$	$(-a,b)$	$(0,0)$	After

Transform to S' where 1 has $v_x = 0$



use relativistic velocity transformations

$$v_x' = \frac{v_x - u}{1 - \frac{v_x u}{c^2}} \qquad v_y' = \frac{v_y}{\gamma\left(1 - \frac{v_x u}{c^2}\right)}$$

$$u = a \;,\quad \beta = \frac{a}{c} \;,\quad \gamma = \frac{1}{\sqrt{1 - \frac{a^2}{c^2}}}$$

$\vec{v}_1$	$\vec{v}_2$	$\vec{p}_1 + \vec{p}_2$	
$\left(0, \frac{b}{\gamma(1-\beta^2)}\right)$	$\left(\frac{-2a}{1+\beta^2}, \frac{-b}{\gamma(1+\beta^2)}\right)$	$\left(\frac{-2ma}{1+\beta^2}, \frac{2mb\beta^2}{\gamma(1-\beta^4)}\right)$	before
$\left(0, \frac{-b}{\gamma(1-\beta^2)}\right)$	$\left(\frac{-2a}{1+\beta^2}, \frac{b}{\gamma(1+\beta^2)}\right)$	$\left(\frac{-2ma}{1+\beta^2}, \frac{-2mb\beta^2}{\gamma(1-\beta^4)}\right)$	After

classical momentum Not conserved!

make a new definition of $\vec{p}$ such that

1. goes to $\vec{p} = m\vec{v}$ at small $\vec{v}$
2. Is conserved in all reference frames

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try $\vec{p} \equiv m\vec{v}\gamma$ $\quad \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$

$$v_1^2 = \frac{b^2}{\gamma_0^2(1-\beta_0^2)^2} \qquad v_2^2 = \frac{4a^2}{(1+\beta^2)^2} + \frac{b^2}{\gamma^2(1+\beta^2)^2} = \frac{4a^2\gamma_0^2+b^2}{\gamma_0^2(1+\beta^2)^2}$$

$$P_{1y} = \frac{mb}{\gamma_0(1-\beta_0^2)} \cdot \frac{1}{\sqrt{1-\frac{b^2}{c^2\gamma_0^2(1-\beta_0^2)^2}}}$$

$$P_{2y} = \frac{-mb}{\gamma(1+\beta_0^2)} \cdot \frac{1}{\sqrt{1-\frac{4a^2\gamma_0^2+b^2}{c^2\gamma_0^2(1+\beta_0^2)^2}}}$$

before = - after

if $P_{1y} + P_{2y} = 0$ then $\vec{p} \equiv m\vec{v}\gamma$ is conserved

$\frac{mb}{\gamma_a}$ terms kept out

$$P_{1y} = \left[(1-\beta^2)^2 - \frac{b^2}{c^2\gamma^2} \right]^{-1/2} \frac{mb}{\gamma_a}$$

$$P_{2y} = \left[(1+\beta^2)^2 - \frac{4a^2\gamma^2 + b^2}{c^2\gamma^2} \right]^{-1/2} - \frac{mb}{\gamma_a}$$

$$P_{1y} = \left[\frac{1}{c^2\gamma^2} \left(c^2\gamma^2 (1 - 2\beta^2 + \beta^4) - b^2 \right) \right]^{-1/2} \frac{mb}{\gamma_a}$$

$$P_{2y} = \left[\frac{1}{c^2\gamma^2} \left(c^2\gamma^2 (1 + 2\beta^2 + \beta^4) - \underbrace{4a^2\gamma^2}_{4\beta^2 c^2\gamma^2} + b^2 \right) \right]^{-1/2} - \frac{mb}{\gamma}$$

so $P_{1y} = -P_{2y}$

$P_{1y} + P_{2y} = 0$ before and after

so $\vec{p} \equiv m\vec{v}\gamma$ is conserved under transformation of frames

Relativistic Energy

Similar to momentum, define E such that

1. is conserved under transformations
2. goes to classical definition at small v.

2: Try $E \equiv \gamma m c^2$

$$E = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} mc^2$$

for small v, $v \ll c$, use binomial approximation

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$E \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) mc^2 = mc^2 + \frac{1}{2}mv^2$$

Kinetic Energy

rest mass energy

HW: use our example of 2 colliding balls to show $\sum_i \gamma m_i c^2$ is conserved during the collision in both reference frames, S and S'

HW: 4.13 4.48