$$\psi(r) = C\, e^{-r/\delta}$$

$$\delta = a_0$$

Find normalization constant C

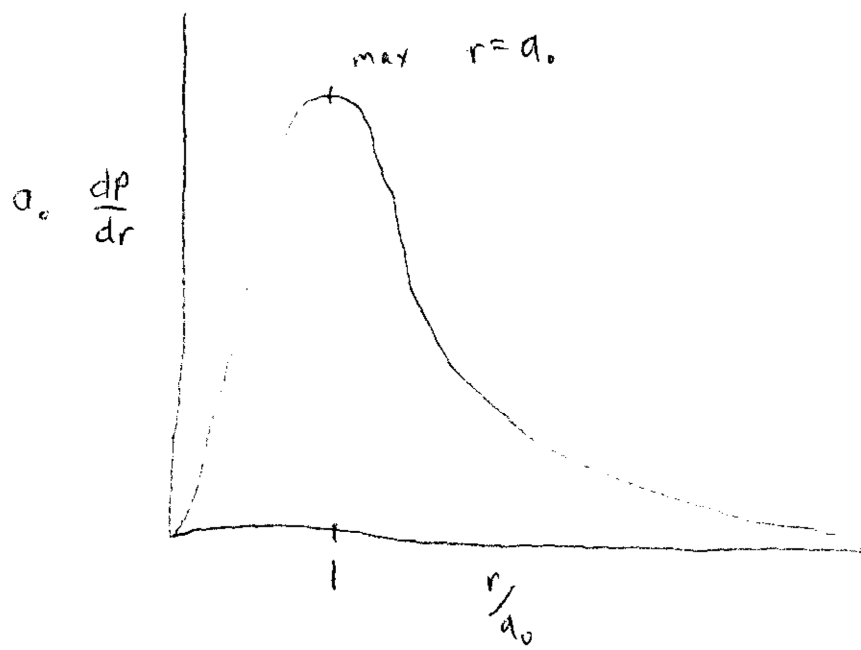
$$\iiint_V dV\, |\psi|^2 = \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \int_0^{\infty} dr\; r^2\, |\psi|^2$$

$$= 4\pi \int_0^{\infty} dr\; r^2\, C^2\, e^{-2r/a_0} = 1$$

$$C = \frac{1}{\sqrt{\pi}\, a_0^{3/2}}$$

$$\psi(r) = \frac{1}{\sqrt{\pi}\, a_0^{3/2}}\, e^{-r/a_0}$$

$$\frac{dP}{dr} = 4\pi r^2\, |\psi|^2 = \frac{4r^2}{a_0^3}\, e^{-2r/a_0}$$



Hydrogen Atom General Solution

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{ke^2}{r}\psi = E\psi$$

$$\nabla^2\psi = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial\phi^2}\right]$$

separation of variables

$$\psi(r,\theta,\phi) = R(r)\,P(\theta)\,F(\phi)$$

$$\frac{\partial\psi}{\partial r} = P(\theta)F(\phi)\frac{\partial R(r)}{\partial r}$$

$$\frac{\partial\psi}{\partial\theta} = R(r)F(\phi)\frac{\partial P(\theta)}{\partial\theta}$$

$$\frac{\partial^2\psi}{\partial\phi^2} = R(r)\,P(\theta)\frac{\partial^2 F(\phi)}{\partial\phi^2}$$

Schrodinger equation becomes

$$\left[\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2mEr^2}{\hbar^2} + \frac{2mke^2r}{\hbar}\right] \leftarrow$$

is a function only of r, must be a constant, $\equiv C_r$

$$+\left[\frac{1}{P\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) + \frac{1}{F\sin^2\theta}\frac{d^2F}{d\phi^2}\right] = 0$$

$$\frac{1}{F}\frac{d^2F}{d\phi^2} = -C_r\sin^2\theta - \frac{\sin\theta}{P}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) \equiv C_\phi$$

both sides are a function of only one variable,

so they must be constants

$$\frac{1}{R}\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) + \frac{2mEr^2}{\hbar^2} + \frac{2mke^2 r}{\hbar^2} \equiv C_r$$

$$\frac{1}{F}\frac{d^2F}{d\phi^2} \equiv C_\phi$$

$$C_\phi = -C_r \sin^2\theta - \frac{\sin\theta}{P}\frac{d}{d\theta}\left(\sin\theta \frac{dP}{d\theta}\right)$$

separated differential equations

The equations are coupled through C_r and C_ϕ

The solutions place restrictions on C_ϕ, C_r

$$C_r = l(l+1) \qquad l = 0, 1, 2 \ldots$$

$$C_\phi = -m_l^2 \qquad m_l = -l, -l+1, \ldots l-1, l$$

from radial equation

$$E_n = -\frac{m_e (ke^2)^2}{2\hbar^2}\cdot\frac{1}{n^2} = -\frac{\alpha^2 mc^2}{2n^2}$$

$$n = l+1, l+2, l+3 \ldots$$

or, for a certain n, or E_n,

$$l = 0, 1, 2 \ldots n-1$$

HW #4

8.3 8.7 8.12 8.13