Total Angular Momentum

\[ \vec{J} = \vec{L} + \vec{S} \]

\[ S = \sqrt{s(s+1)} \hbar \]

\[ s = \frac{1}{2} \]

\[ S = \frac{\sqrt{3}}{2} \hbar \]

\[ S_z = m_s \hbar , \quad m_s = \pm \frac{1}{2} \]

\[ J = \sqrt{j(j+1)} \hbar \]

\[ j = |l-s|, |l-s|+1, \ldots, |l+s| \]

\[ J_z = m_J \hbar \]

spectroscopic notation \[ n \ell s \] \( \ell \) express \( s, p, d, f, \ldots \)
Hydrogen Ground state \( 1s_{\frac{1}{2}} \)

\[ l = 0 \]
\[ s = \frac{1}{2} \]
\[ j = |l-s|, |l-s|+1, \ldots |l+s| \]

\[ j = \frac{1}{2} \]
\[ J = \sqrt{j(j+1)} \frac{\hbar}{2} = \frac{\sqrt{13}}{2} \hbar \]
\[ J_z = \pm \frac{\hbar}{2} \]

p-state

\( 2p_{\frac{1}{2}}, 2p_{\frac{3}{2}} \) - energy level doesn't affect angular momentum

\[ l = 1, s = \frac{1}{2} \]
\[ j = \frac{1}{2}, \frac{3}{2} \]
\[ J = \sqrt{\frac{13}{2}} \hbar, \sqrt{15} \frac{\hbar}{2} \]
\[ J_z = -\frac{\hbar}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2} \hbar \]
Spin-Orbit: Fine structure

The electron feels a magnetic field from the proton—the proton is a moving charge in the electron's rest frame.

\[ \Delta E = -\vec{\mu}_e \cdot \vec{B}_{int} = \frac{e}{2m} \gamma \vec{S} \cdot \vec{B}_{int} = C \vec{S} \cdot \vec{L} \]

Use classical physics and Bohr model to estimate \( B_{int} \)

\[ B = \left( \frac{4\pi k}{\varepsilon^2} \right) \left( \frac{I}{2r} \right) \]

\[ I = \frac{eV}{2\pi r} \]

\[ B = \frac{keV}{2\pi r^2} \]

\[ \Delta E \approx \alpha \left( \frac{V}{c} \right) \left( \frac{\hbar^2}{2mr^2} \right) \]

for \( n=2 \)

\[ \frac{V}{c} = \frac{1}{2}, \quad r = 4a_0 = \frac{4\hbar c}{\alpha mc^2} \]

\[ \Delta E \approx \alpha^4 \frac{mc^2}{64} \]

\( \alpha \) - Fine structure constant

This gives an energy level splitting of \( 2p \) state

\[ E(2p_{3/2}) = E_2 + \Delta E \]

\[ E(2p_{1/2}) = E_2 - \Delta E \]
Hyperfine structure

In hydrogen atom ground state

$ls, \ell = 0, L = 0$, no spin-orbit coupling

Proton has 'spin' - intrinsic angular momentum $\mathbf{S} \sim \frac{\mu}{r^3}$ $\mu$ - magnetic moment of the proton

The field is weak, but gives rise to an energy splitting of $m_s = \frac{1}{2}, m_s = -\frac{1}{2}$

$m_s = \frac{1}{2} \rightarrow m_j = \frac{1}{2}$

$\Delta E$ is equal to energy of $\lambda = 21 \text{ cm photon}$
Atomic Selection Rules

Photon has spin, $s = 1$

$m_s = +1, -1$

Conservation of total angular momentum places limits on allowed transitions:

$\Delta j = \pm 1$

$\Delta m_j = 0, \pm 1$

$\Delta ms = 0$

$\Delta j^z = 0, \pm 1$

$j^z = 0 \rightarrow j^z = 0$ is forbidden