Total Angular Momentum

$$\vec{J} = \vec{L} + \vec{S}$$

$$S = \sqrt{s(s+1)}\,\hbar$$

$$s = \frac{1}{2}$$

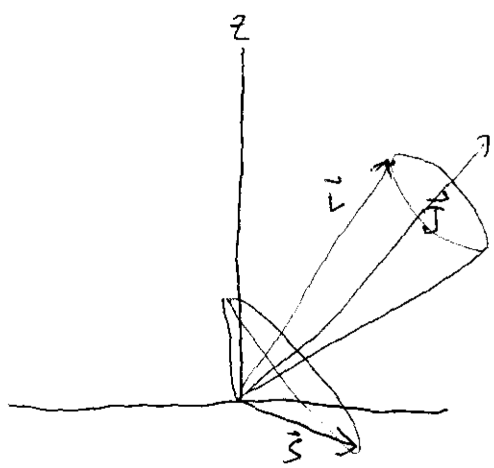
$$S = \frac{\sqrt{3}}{2}\hbar$$

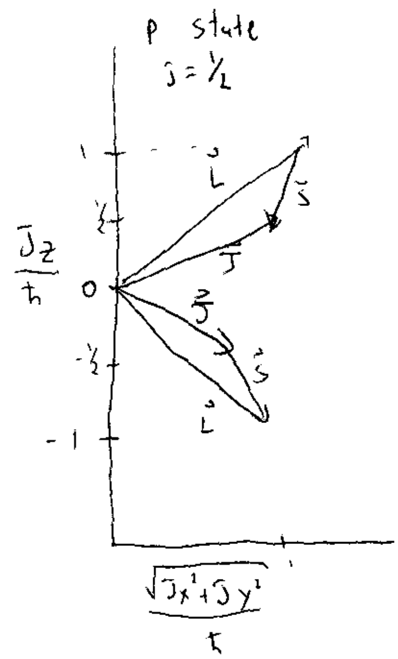
$$S_z = m_s \hbar \quad , \quad m_s = \pm \frac{1}{2}$$

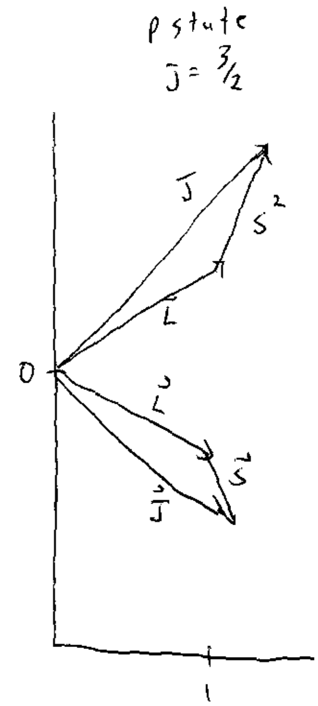
$$J = \sqrt{j(j+1)}\,\hbar$$

$$j = |l-s|, |l-s|+1, \dots |l+s|$$

$$J_z = m_j \hbar$$







Spectroscope notation $n l_j$, l expressed s, p, d, f, ...

Hydrogen Ground state $1s_{1/2}$

$l = 0$

$s = \frac{1}{2}$

$j = |l-s|, |l-s|+1, \dots |l+s|$

$j = \frac{1}{2}$

$J = \sqrt{j(j+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$

$J_z = \pm \frac{\hbar}{2}$

p-state

$2p_{1/2}$, $2p_{3/2}$ - energy level doesn't affect angular momentum

$l = 1$, $s = \frac{1}{2}$

$j = \frac{1}{2}, \frac{3}{2}$

$J = \frac{\sqrt{3}}{2}\hbar, \frac{\sqrt{15}}{2}\hbar$

$J_z = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\ \hbar$

Spin-Orbit : Fine structure

The electron feels a magnetic field from the proton - the proton is a moving charge in the electrons rest frame,

$$\Delta E = -\vec{\mu}_s \cdot \vec{B}_{int} = \frac{e}{2m} g \vec{S} \cdot B_{int} = C \vec{S} \cdot \vec{L}$$

Use classical physics and Bohr model to estimate B_{int}

$$B = \left(\frac{4\pi k}{c^2}\right)\left(\frac{I}{2r}\right)$$

$$I = \frac{ev}{2\pi r}$$

$$B = \frac{kev}{c^2 r^2}$$

$$\Delta E \approx \alpha \left(\frac{v}{c}\right)\left(\frac{\hbar^2}{2mr^2}\right)$$

for $n=2$

$$\frac{v}{c} = \frac{\alpha}{2} \quad , \quad r = 4a_0 = \frac{4\hbar c}{\alpha mc^2}$$

$$\Delta E \approx \alpha^4 \frac{mc^2}{64}$$

α - Fine structure constant

This gives an energy level splitting of $2p$ state

$$E(2p_{3/2}) = E_2 + \Delta E$$
$$E(2p_{1/2}) = E_2 - \Delta E$$

Hyper fine structure

In hydrogen atom ground state

$1s_{1/2}$ $\quad l=0$, $L=0$, no spin-orbit coupling

proton has 'spin' - intrinsic angular momentum

$\vec{B} \sim \frac{\vec{\mu}}{r^3}$ $\quad \vec{\mu}$ - magnetic moment of the proton

The field is weak, but gives rise

to an energy splitting of $m_j = \frac{1}{2}$, $m_j = -\frac{1}{2}$

$m_j = \frac{1}{2} \longrightarrow m_j = -\frac{1}{2}$

ΔE is equal to energy of

$\lambda = 21$ cm photon

Atomic Selection Rules

photon has spin, $s = 1$

$m_s = +1, -1$

Conservation of total angular momentum places limits on allowed transitions.

$\Delta l = \pm 1$

$\Delta m_l = 0, \pm 1$

$\Delta m_s = 0$

$\Delta j = 0, \pm 1$

$j = 0 \rightarrow j = 0$ is forbidden