Momentum - Energy Relationships

$$\vec{p} \equiv m\vec{v}\gamma$$

$$E \equiv mc^2\gamma$$

$$E^2 = p^2c^2 + (mc^2)^2 \qquad \frac{v}{c} = \frac{pc}{E}$$

for a photon $m = 0$, $v = c$

$$E = pc$$

$$E = hf = \frac{hc}{\lambda}$$

Force

$$\vec{F} = \frac{d\vec{p}}{dt}$$

now use relativistic momentum

$$\vec{p} = \gamma m \vec{v}$$

$$\vec{F} = m \frac{d(\gamma \vec{v})}{dt} = m\gamma \frac{d\vec{v}}{dt} + m\vec{v}\frac{d\gamma}{dt}$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\frac{d\gamma}{dt} = \left(-\frac{1}{2}\right)\left(-\frac{2v}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2} = \frac{\gamma^3 v}{c^2}\frac{dv}{dt}$$

$$\vec{F} = m\gamma \frac{d\vec{v}}{dt} + m\vec{v}\frac{\gamma^3 v}{c^2}\frac{dv}{dt}$$

use "trick" to get only $\vec{F}$ and $\frac{d\vec{v}}{dt}$ in equation

$$\vec{F}\cdot\vec{v} = m\gamma\left(\frac{d\vec{v}}{dt}\cdot\vec{v}\right) + \frac{m\gamma^3 v}{c^2}\frac{dv}{dt}(\vec{v}\cdot\vec{v})$$

$v\frac{dv}{dt}$ $\qquad$ v^2

Solve for $\frac{dv}{dt} = \frac{\vec{F}\cdot\vec{v}}{mv\gamma^3}$

$$\vec{F} = m\gamma\frac{d\vec{v}}{dt} + \frac{\vec{v}(\vec{F}\cdot\vec{v})}{c^2}$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = \frac{\vec{F} - \vec{\beta}(\vec{F}\cdot\vec{\beta})}{m\gamma}$$

$$\vec{a} = \frac{\vec{F}}{m\gamma} - \frac{\vec{\beta}(\vec{F}\cdot\vec{\beta})}{m\gamma}$$

resistance to $\vec{a}$ in direction of velocity

same as classical but with 'relativistic mass' $m\gamma$

Charged particle in magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

no component of force in direction of $\vec{v}$

so $\vec{a} = \frac{d\vec{v}}{dt} = \frac{\vec{F}}{m\gamma} = \frac{q\vec{v}\times\vec{B}}{m\gamma}$

for circular orbit

$$a = \frac{v^2}{r}$$

direction $\vec{a}$ is always towards center

magnitude of $\vec{v}\times\vec{B}$ is vB

$$\frac{v^2}{r} = \frac{qvB}{\gamma m}$$

$$p = \gamma m v = qrB$$

The Probability Distribution

$\frac{dP}{dx}$ - probability distribution, relative probability for an event to occur for diffrent values of x

$$\int_{-\infty}^{\infty} dx \ \frac{dP}{dx} = 1$$

mean value $\langle x \rangle = \int_{-\infty}^{\infty} x \ dx \ \frac{dP}{dx}$

note $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \ dx \ \frac{dP}{dx}$

$$\langle x \rangle^2 = \left[\int_{-\infty}^{\infty} x \ dx \ \frac{dP}{dx} \right]^2$$

$\langle x^2 \rangle \neq \langle x \rangle^2$ in general

Standard Deviation

$$\sigma \equiv \left[\langle (x - \langle x \rangle)^2 \rangle \right]^{\frac{1}{2}}$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \langle x \rangle)^2 dx \ \frac{dP}{dx}$$

variance

Example

$-\infty : 0 \qquad \frac{dP}{dx} = 0$

$0 : 1 \qquad \frac{dP}{dx} = 1$

$1 : \infty \qquad \frac{dP}{dx} = 0$

check normalization

$$\int_0^1 1\,dx = 1 \qquad \text{OK.}$$

$$\langle x \rangle = \int_0^1 x\,dx = \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2}$$

$$\sigma^2 = \int_0^1 \left(x - \tfrac{1}{2}\right)^2 dx = \int_0^1 dx\left(x^2 - x + \tfrac{1}{4}\right)$$

$$= \left.\left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{x}{4}\right)\right|_0^1 = \frac{1}{2}$$

$$\sigma = \sqrt{\frac{1}{12}}$$

Binomial Distribution

n: number of trials

p: probability of success on each trial

x: number of successes ~~after~~ in the experiment

$$\frac{dp}{dx} = f(x) = \frac{n!\, p^{x} (1-p)^{n-x}}{x!\,(n-x)!}$$

In class problem

given $n = 5$ (equivalent to 5 coin flips)
$p = 0.5$

~~what~~ what is chance to get 0, 1, 2 or 5 positive events?