$$\frac{d\sigma}{d\cos\theta} = 2\pi Z^2 \alpha^2 \left(\frac{\hbar c}{E_K}\right)^2 \frac{1}{(1-\cos\theta)^2}$$

$$b = \frac{K q_1 q_2}{m v^2} \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{2 K Z e^2}{m v^2} \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

↑ for α scattering

cross section for α scattering at angles greater than θ_{min}

$$\sigma = \int_0^{b_{max}} db \, \frac{d\sigma}{db} = 2\pi \int_0^{b_{max}} b \cdot db = \pi b_{max}^2$$

$$b_{max} = \frac{K Z e^2}{E_K} \sqrt{\frac{1+\cos\theta_{min}}{1-\cos\theta_{min}}}$$

$$\sigma = \pi Z^2 \left(\frac{K e^2}{E_K}\right)^2 \left(\frac{1+\cos\theta_{min}}{1-\cos\theta_{min}}\right)$$

In class problem

A beam of α- particles ($E_K = 5$ MeV) are incident on a silver nucleus ($Z=47$)

what must the flux be to see scattering into angles greater than $\pi/2$ at 1 event/second?

Solution

$$\sigma = \pi Z^2 \left(\frac{ke^2}{E_K}\right)^2 = \pi (47)^2 \left(\frac{1.44\ eV \cdot 10^{-9} m}{5 \times 10^6\ eV}\right) = 5.8\ b$$

$$1b = 10^{-28}\ m^2$$

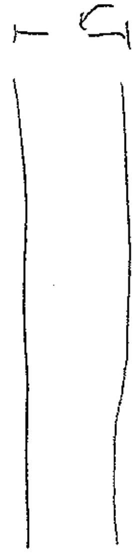
$$\sigma = \frac{R_s}{\Phi} \quad \begin{matrix} \leftarrow \text{Scattering Rate} \\ \leftarrow \text{flux} \end{matrix}$$

$$\Phi = \frac{R_s}{\sigma} = \frac{R_s}{\pi Z^2}\left(\frac{E_K}{ke^2}\right)^2 = 1.7 \times 10^{27}\ m^{-2} s^{-1}$$

$$= 17 \times 10^6\ \mathring{A}^{-2}\ s^{-1}$$

Scattering by thin foils

thickness $= L$



foil is made up of Atoms with

atomic mass A

density ρ

number density $= \frac{n}{V} = \frac{N_A \rho}{A(10^{-3}\ \text{kg})}$

↑ or grams, since N_A is related to grams

(N_A = number of atoms in 12 g of C^{12})

effective flux

$$\Phi = \frac{R_i L n}{V} = \frac{R_i L N_A \rho}{A(10^{-3}\ \text{kg})}$$

$$\sigma = \frac{R_s}{\Phi} = \frac{R_s A(10^{-3}\ \text{kg})}{R_i L N_A \rho}$$

No longer need to know flux, just incident rate of α-particles, for 5 MeV α- particles incident on silver

$\sigma = 5.8\ b$ for scattering events larger than $\pi/2$

for a silver foil of $1\mu m$ thickness

what incident rate do we need to see

1 event/s ?

$$R_i = \frac{R_s\, A\,(10^{-3}\ kg)}{\sigma\, L\, N_A\, \rho}$$

$\rho = 1.05 \times 10^4\ kg/m^3$

$A = 108$

$$R_i = \frac{(1\ s^{-1})(108)(10^{-3}\ kg)}{(5.8 \times 10^{-28}\ m^2)(10^{-6}\ m)(6.02 \times 10^{23})(1.05 \times 10^4\ kg/m^3)}$$

$R_i = 30{,}000$ α -particles/second