Temperature

0^{th} law of thermodynamics

There exists a use full quantity, <u>temperature</u>, which is a measure of the equilibrium state.

when two systems are in equilibrium ~~[illegible]~~, and there is no net energy flow between them, they have the same temperature.

Define temperature

$$\langle E_K \rangle \equiv \frac{3}{2} KT \quad \text{for an ideal gas}$$

$K = 8.617 \times 10^{-5}$ eV/K - Boltzmann constant

and T is in Kelvin scale

for N_2 molecule

$$E_K = \frac{1}{2} m v^2$$

$$\left\langle \frac{m v^2}{2} \right\rangle = \frac{3}{2} KT$$

$$\sqrt{\langle v^2 \rangle} \sim 510 \ m/s$$

Ideal gas law

$$\frac{F}{A} = \frac{\left(\frac{\Delta p}{\Delta t}\right)}{A}$$

from a single molecule $\Delta p = 2mv_x$

$$\frac{F}{A} = \frac{2mv_x}{A\,\Delta t}$$

only particles within $v_x \Delta t$ can hit within Δt

fraction of total volume that can hit area A

is $\frac{v_x \Delta t \cdot A}{V}$

Only $\frac{1}{2}$ are moving in the correct direction

$$H = \frac{v_x \Delta t\, A}{2V}$$

$$P_s = H\left(\frac{F}{A}\right) = \frac{mv_x^2}{V}$$

$$P = \frac{Nm\langle v_x^2\rangle}{V}$$

$$\langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle = 3\langle v_x^2\rangle$$

$$P = \frac{1}{3}\frac{Nm\langle v_x^2\rangle}{V} \qquad PV = \frac{2}{3}N\left\langle\frac{mv^2}{2}\right\rangle$$

$\equiv \frac{3}{2}kT$

$$PV = NkT$$

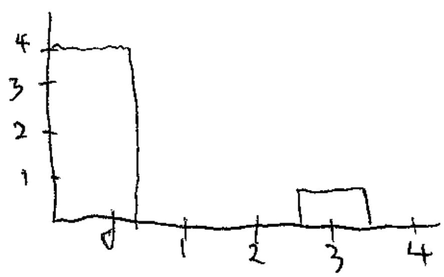
Maxwell - Boltzmann Distribution

If the total energy of a system is divided among a large number of particles, all possible divisions of energy are equally likely.

$$f_{MB} = C\, e^{-E/kT}$$

Example: total Energy = 3
5 particles

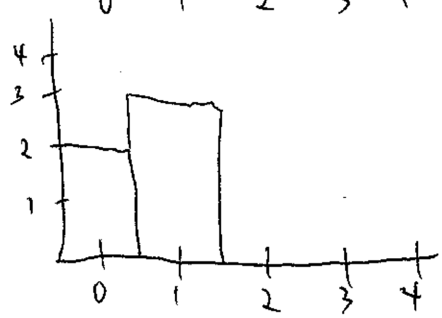
Combinations



5

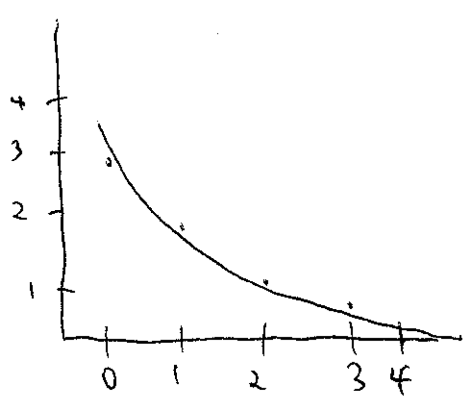
$\frac{\Delta N}{\Delta E}$

20



10

$\frac{dN}{dE}$



Another argument for exponential distribution

2 particles, total Energy = 10

$E_1 + E_2 = 10$

$P(E_1)\,P(E_2)$ - probability of particle 1 having $E = E_1$
and particle 2 having $E = E_2$

if all divisions of energy are equally likely

$$P(0)\,P(10) = P(1)\,P(9) = P(2)\,P(8)$$

$$P(E_1)\,P(E_2) = f(E_1 + E_2)$$

only exponential can do this $\quad f = Ce^{-E/KT}$

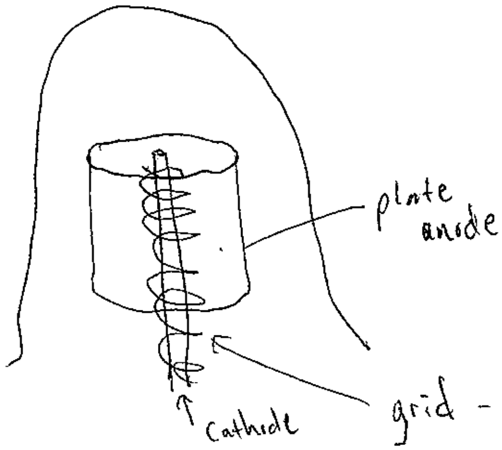
KT is denominator such that

$$\langle E \rangle = KT$$

Correct behavior for ideal gass law is restored by considering density of states.

Vacuum Tubes

$$J = C\,(kT)^2\, e^{-\Phi/kT}$$



grid - can modulate current flow

Density of states

$$\rho(E) \equiv \frac{dn_s}{dE}$$ - density of states

$n_s = \frac{N_s}{V}$ is number of states per volume with energy between E and E+dE

$$\frac{df}{dE} \propto \rho(E)\, e^{-E/kT}$$ number of occupied states per volume with energy between E and E+dE

for an ideal gas

$$\rho(E) = \left(\frac{N}{V}\right)(2\pi)\left(\frac{1}{\pi kT}\right)^{3/2}\sqrt{E}$$

maxwell speed distribution

$$\frac{d^3f}{dv_x\, dv_y\, dv_z} = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}$$

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$dv_x\, dv_y\, dz = (dv)(v\,d\theta)(v \sin\theta\, d\phi)$$

$$\frac{d^3f}{dv\, d\theta\, d\phi} = v^2 \sin\theta \frac{d^3f}{dv_x\, dv_y\, dv_z}$$

$$\frac{df}{dv} = \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi \; v^2 \frac{d^3f}{dv_x\, dv_y\, dv_z}$$

$$\frac{df}{dv} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

Energy Distribution

$$E = \frac{1}{2} m v^2$$

$$dE = m v \, dV$$

$$\frac{df}{dE} = \frac{dV}{dE} \cdot \frac{df}{dV} = \frac{4\pi}{mV} \left(\frac{m}{2\pi kT} \right)^{3/2} V^2 e^{-mv^2/2kT}$$

$$\frac{df}{dE} = 2\pi \left(\frac{1}{\pi kT} \right)^{3/2} \sqrt{E} \, e^{-E/kT}$$

$$\frac{dn}{dE} = \frac{N}{V} \frac{df}{dE}$$

$$\frac{dn}{dE} = \rho(E) \, f_{MB}$$

$$\rho(E) = \left(\frac{N}{V} \right) (2\pi) \left(\frac{1}{\pi kT} \right)^{3/2} \sqrt{E}$$