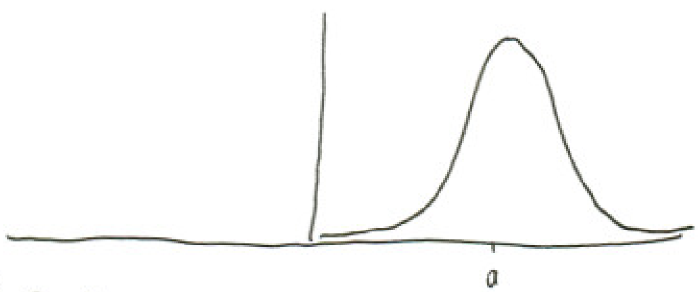
Gaussian distribution function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}}$$

large 'n' limit of binomial distribution

also called 'normal' distribution



$\langle x \rangle = a$

$\sigma = \sigma$

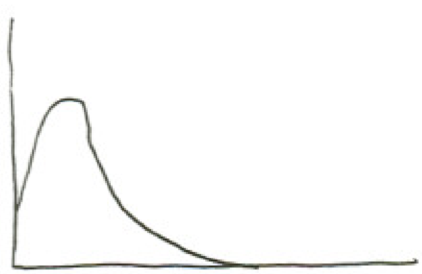
Poisson distribution function

$$f(x) = \frac{e^{-a} a^x}{x!}$$

large-n, small-p limit of binomial distributions

$\langle x \rangle = a$

$\sigma = \sqrt{a}$



De broglie wavelength

from compton scattering and photoelectric effect

$E = \frac{hc}{\lambda}$ for photons

$E = pc$. from relativity

so $\lambda = \frac{h}{p}$

If EM radiation can be treated as particles, why not treat particles as waves? (1924)

Bohr model of Hydrogen use quantization condition

$$L = pr = \frac{nh}{2\pi}$$

$\uparrow$
angular momentum

if $n\lambda_e = 2\pi r$ - integer number of wavelength in one orbit

$$\lambda_e = \frac{h}{p}$$

Waves

angular frequency

$$w = 2\pi f$$

wave number

$$K = \frac{2\pi}{\lambda}$$

wave equation

$$\Delta F = \nabla^2 F = \frac{\partial^2 F}{\partial x} + \frac{\partial^2 F}{\partial y} + \frac{\partial^2 F}{\partial z} = \frac{1}{v^2}\frac{\partial^2 F}{\partial t}$$

'Laplacian' operator

$$v = f\lambda = \frac{w}{K}$$

Compact way to write solution to wave equation

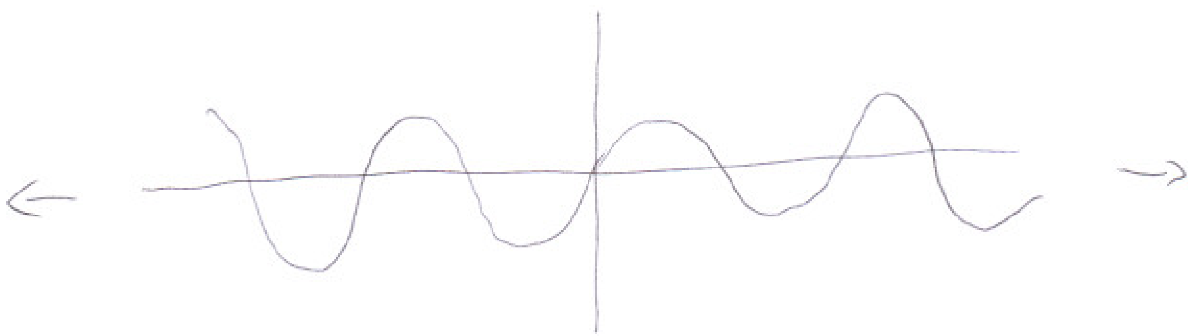
$$F(x,t) = A\,e^{i(Kx - wt)}$$

$$= A\left[\cos(Kx - wt) + i\sin(Kx - wt)\right]$$

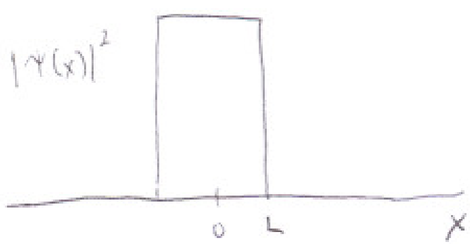
Fourier Analysis

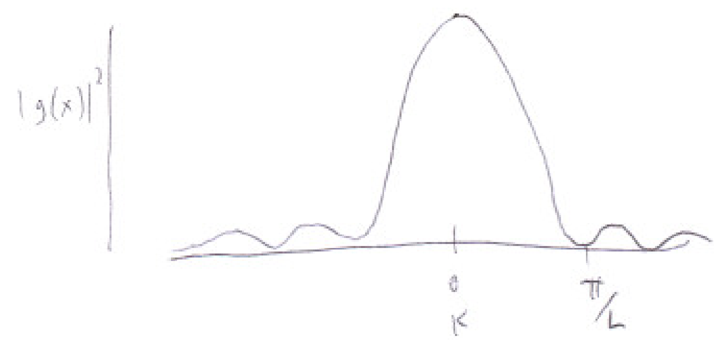
consider the spatial extent of a wave form
(neglect time dependence)

A 'plane wave' will be made up of only one wavelength, or K value



more complicated shapes can be made by adding waves with different K values





this shape can be made by adding waves with relative strengths. this is the wave number distribution or 'Fourier Transform'

If a wave form is periodic.

$$F(x) = \sum_{n=-\infty}^{n=\infty} A_n e^{inkx}$$

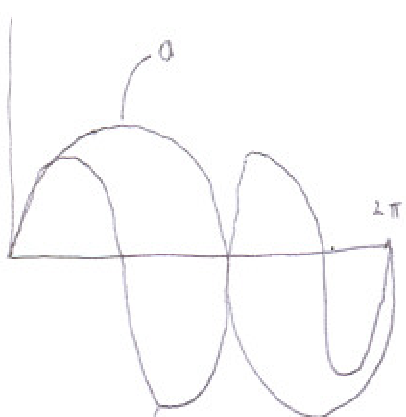
A_n are fourier coefficients

$$A_n = \frac{K}{2\pi} \int_0^{2\pi/k} dx \; F(x) e^{-inkx}$$

we can pick out the coefficients this way since waves with diffrent values of n are 'orthagonal' meaning

$$\int_0^{2\pi/k} dx \, e^{+inkx} e^{-imkx} = 0 \quad \text{unless} \quad n = m$$

example:



$a = \sin(x)$

$b = \sin(2x)$

$$\int_0^{2\pi} \sin(x) \sin(2x) \, dx$$

$$= \int_0^{2\pi} 2 \sin^2(x) \cos(x) \, dx$$

$u = \sin(x)$

$du = \cos(x) dx$

$$\rightarrow \int 2x^2 dx = \frac{2x^3}{3} : \left. \frac{2 \sin^3(x)}{3} \right|_0^{2\pi} = 0$$

$$A_m = \frac{k}{2\pi} \int_0^{2\pi/k} dx \sum_{n=-\infty}^{n=\infty} A_n e^{inkx} e^{-imkx}$$

only nonzero when $n = m$

$$A_m = \frac{k}{2\pi} \int_0^{2\pi/k} A_n e^0 = A_n$$

Can extend to non-periodic 'wave packets'

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, g(k)\, e^{-ikx}$$

$$g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, \Psi(x)\, e^{ikx}$$