Gaussian distribution function

\[ f(x) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}} \]

Large 'n' limit of binomial distribution

also called 'normal' distribution

\[ \langle x \rangle = a \]
\[ \sigma = \sigma \]

Poisson distribution function

\[ f(x) = \frac{e^{-\lambda} \lambda^x}{x!} \]

Large 'n', small 'p' limit of binomial distributions

\[ \langle x \rangle = a \]
\[ \sigma = \sqrt{a} \]
De Broglie wavelength

from Compton scattering and photoelectric effect

\[ E = \frac{hc}{\lambda} \quad \text{for photons} \]

\[ E = pc \quad \text{from relativity} \]

so \[ \lambda = \frac{h}{p} \]

If EM radiation can be treated as particles, why not treat particles as waves? (1924)

Bohr model of hydrogen use quantization condition

\[ L = pr = \frac{nh}{2\pi} \]

\[ \uparrow \]

Angular momentum

If \[ n \lambda_e = 2\pi r \quad \text{integer number of wavelengths in orbit} \]

\[ \lambda_e = \frac{h}{p} \]
Waves

Angular frequency
\[ \omega = 2\pi f \]

Wave number
\[ K = \frac{2\pi}{\lambda} \]

Wave equation
\[ \Delta F = \nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = \frac{i}{\nu} \cdot \frac{\partial^2 F}{\partial t^2} \]

"Laplacian" operator
\[ \nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} \]

Compact way to write solution to wave equation
\[ F(x, t) = A e^{i(kx - \omega t)} \]
\[ = A \left[ \cos(kx - \omega t) + i \sin(kx - \omega t) \right] \]
Fourier Analysis

Consider the spatial extent of a waveform (neglect time dependence).

A 'plane wave' will be made up of only one wavelength, or \( k \) value.

More complicated shapes can be made by adding waves with different \( k \) values.

\[ |Y(x)|^2 \quad |g(x)|^2 \]

\[ o \quad L \quad x \quad 0 \quad \pi/L \]

This shape can be made by adding waves with relative strengths. This is the wave number distribution or 'Fourier Transform.'
If a waveform is periodic:

\[ F(x) = \sum_{n=-\infty}^{\infty} A_n e^{i n k x} \]

\[ A_n = \frac{1}{2\pi} \int_{0}^{2\pi} F(x) e^{-i n k x} \, dx \]

we can pick out the coefficients this way since waves with different values of \( n \) are orthogonal meaning

\[ \int_{0}^{2\pi} e^{-i n k x} e^{-i m k x} \, dx = 0 \text{ unless } n = m \]

Example:

\[ a = \sin(x) \]
\[ b = \sin(2x) \]

\[ \int_{0}^{2\pi} \sin(x) \sin(3x) \, dx \]

\[ = \int_{0}^{2\pi} 2 \sin^3(x) \cos(x) \, dx \]

\[ = 2 \sin^3(x) \cos(x) \bigg|_{0}^{2\pi} \]

\[ = 0 \]
\[ A_m = \frac{k}{2\pi} \int_0^{2\pi/k} dx \sum_{n=-\infty}^{\infty} A_n e^{i n k x} - i n k x \]

only nonzero when \( n = m \)

\[ A_m = \frac{k}{2\pi} \int_0^{2\pi/k} A_n e^{i n k x} = A_n \]

Can extend to non-periodic 'wave packets':

\[ \Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \ g(k) e^{-ikx} \]

\[ g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \ \Psi(x) e^{ikx} \]