Wave Packets

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, g(k)\, e^{+ikx}$$

$$g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, \psi(x)\, e^{-ikx}$$

$$\frac{dP}{dx} = |\psi(x)|^2$$ - probability of finding a particle per unit distance

A precisely localized particle

$$\frac{dP}{dx} = |\psi(x)|^2 = \lim_{L \to 0} \left[\frac{1}{\sqrt{2\pi L^2}} e^{-(x-a)^2/2L^2} \right] \equiv \delta(x-a)$$

Dirac Delta function

$$\int_{-\infty}^{\infty} f(x)\, \delta(x-a)\, dx = f(a)$$

$$g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, \delta(x-a)\, e^{-ikx} = \frac{1}{\sqrt{2\pi}} e^{-ika}$$

$$|g(k)|^2 = \frac{1}{2\pi}$$ ~ needs all waves from $k = -\infty$ + $k = +\infty$ to make zero width pulse

Gaussian wave packets and the uncertainty principle

$$\Psi(x) = C \; e^{-\frac{x^2}{2L^2}}$$

$\sigma_x = L$ - standard deviation of Gaussian distribution

$$g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \; C \, e^{-x^2/2L^2} \, e^{-ikx}$$

$$= \frac{C}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2L^2}} \; e^{-k^2 L^2/2}$$

$$\sigma_k = \frac{1}{L}$$

$$\sigma_x \sigma_k = 1$$

Probability distributions are $|\Psi(x)|^2$, $|g(k)|^2$

$$|\Psi(x)|^2 = C^2 e^{-x^2/L^2}$$

$$\sigma_x = \frac{L}{\sqrt{2}}$$

$$\sigma_k = \frac{1}{\sqrt{2}L}$$

$$\sigma_x \sigma_k = \frac{1}{2} \qquad\qquad p = \frac{h}{\lambda} = \hbar k$$

$$\sigma_x \sigma_p = \frac{\hbar}{2}$$

Gaussian wave packets give the smallest $\sigma_x \sigma_p$ product. so...

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

particles confined to finite space

have a minimum energy

$$\langle E_k \rangle = \frac{\langle p \rangle^2}{2m}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

↑ goes to zero since ± equaly likely

$$(\Delta p)^2 = \langle p^2 \rangle$$

$$\langle E_k \rangle = \frac{(\Delta p)^2}{2m}$$ ~ sometimes called 'zero point energy'

In class problem:

Estimate the energy - ~~Kin~~ kinetic energy of an electron confined to the size of an atom ~ 0.1 nm ?

Compare this to $E_k = \frac{ke^2}{r}$ ~ the kinetic energy of an electron used in the Bohr model.

HW #2

5.2 , 5.5 5.14 5.19

Due Wed. Jan 31