Particle in a Box

$$\langle E_K \rangle = \frac{(\Delta p)^2}{2m}$$

$$\Delta p = \frac{\hbar}{2\Delta x}$$

$$\langle E_K \rangle > \frac{\hbar^2}{8m(\Delta x)^2}$$

for an electron inside a 0.1 nm box

$$\Delta x = 0.1 nm$$

$$m_e = 0.5 \ MeV/c^2$$

$$\langle E_K \rangle = \frac{(6.6 \times 10^{-16} eV \cdot s)^2}{8 \cdot (0.5 \ MeV/c^2)(.1 nm)^2}$$

$$= \frac{(6.6 \times 10^{-16} eV \cdot s)^2 \cdot (3 \times 10^8 \ m/s)^2}{8 \cdot (0.5 \times 10^6 eV)(10^{-10} m)^2}$$

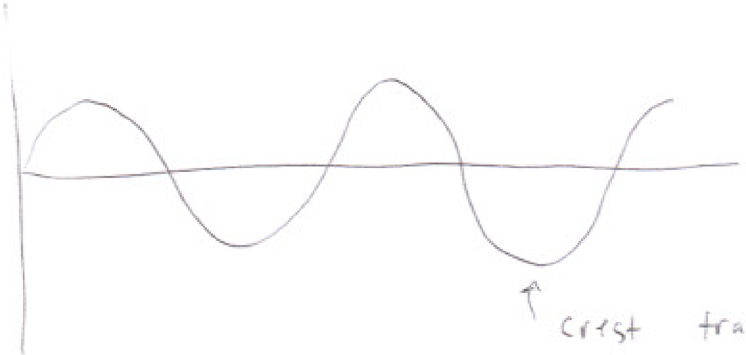
$$= 0.98 \ eV \approx 1 \ eV$$

$$E_K = \frac{Ke^2}{r} = \frac{1.44 \ eV \cdot nm}{0.1 \ nm} = 14 \ nm$$

– bigger, but thats OK by Uncertainty principle

↑

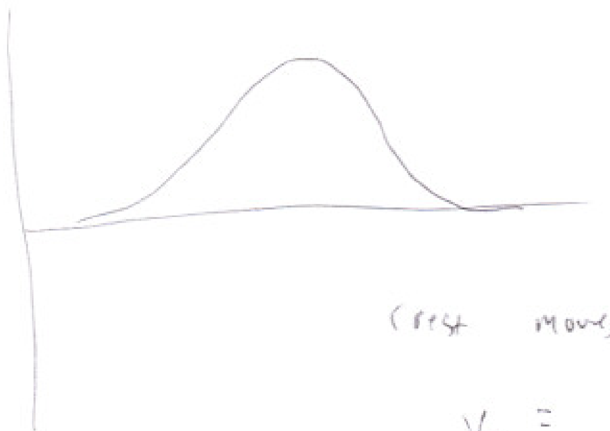
Kinetic energy of circling electron, classical interpretation wave packet motion



- wave mode with single K ~~value~~ value

crest travels at

$v_p = \frac{w}{K}$ - phase velocity



wave packet made from many waves of various k values

crest moves with

$v_g = \frac{dw}{dK}$ group velocity

Extra credit worth 5 points Homework

find the best internet descripton of wave packet movement

Energy - Time Uncertainty

$$E = mc^2 + \frac{p^2}{2m}$$

$$\Delta E = \frac{p\Delta p}{m}$$

$$p = m\frac{\Delta x}{\Delta t}$$

$$\Delta p \Delta x = \frac{m\Delta E}{p} \cdot \frac{p\Delta t}{m} = \boxed{\Delta E \Delta t > \frac{\hbar}{2}}$$

A particles lifetime leads to an uncertainty in its energy.

$\Delta t = \tau$ tau - for exponential decay $\Delta t = \langle t \rangle$

$\Gamma = 2\Delta E$ - full width, half maxima

$$\frac{dN}{dE_0} = \frac{C}{(E_0 - mc^2)^2 + \left(\frac{\Gamma}{2}\right)^2} \quad \text{- Lorentzian}$$

mean mass

number of particles observed to energy E_0

Atomic transitions are about 10 ns,

what is ΔE of photon emitted.

$$\Delta E = \frac{\hbar}{\tau} = \frac{6.6 \times 10^{-16}\, eV \cdot s}{10^{-8}\, s} = 6.6 \times 10^{-8}\, eV$$

in terms of wavelength

$$\Delta\lambda \approx \frac{hc}{\Delta E} = \tau c \frac{h}{\hbar} = 2\pi\, \tau c$$

$$= 2\pi \cdot 10^{-9}\, s \times 3 \times 10^{8}\, m/s = 2\, nm$$

Visible light 400-700 nm

Energy-time uncertainty can lead to

Energy-Non conservation over ~~s...~~ short times.

Schrödinger Equation

The schrödinger Equation is the wave equation for a non-relativistic particle

Time independent

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = (E - mc^2)\Psi(x)$$

Time dependent

$$\psi(x,t) = Ae^{ikx - i\omega t} = \psi(x)e^{-i\omega t}$$

time dependent

$$\psi(x) = Ae^{ikx}$$

putting into time dep. Schrödinger Eq.

$$-\frac{(i)^2 k^2 \hbar^2}{2m} = E - mc^2$$

$$p = \hbar k$$

$$\frac{p^2}{2m} = E - mc^2$$

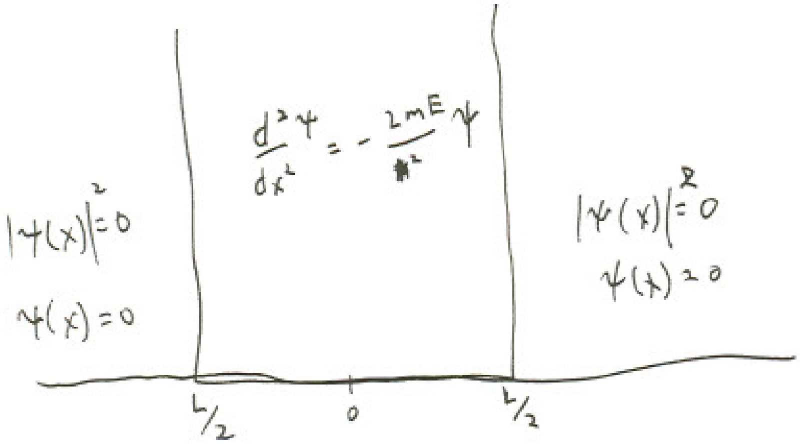
by convention, zero point is mass energy

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi(x)$$

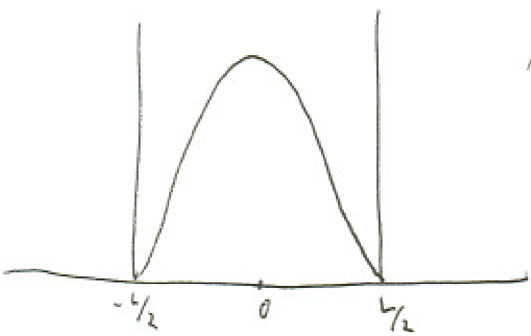
$$E = \frac{p^2}{2m}$$

$\psi(x)$ and $\frac{d\psi(x)}{dx}$ must be continuous

Particle in a Box



Free particle inside
forbidden outside.



$$\psi(x) = A\cos kx$$

$$k = \frac{\pi}{L}$$

longest wavelength, lowest energy

Normalization $\int |\psi(x)|^2 dx = 1$

$$\int_{-L/2}^{L/2} A^2 \cos^2\left(\frac{\pi x}{L}\right) dx = \int_{-L/2}^{L/2} A^2 \sin^2\left(\frac{\pi x}{L}\right) dx$$

so $$\int_{-L/2}^{L/2} A^2 \cos^2\left(\frac{\pi x}{L}\right) dx = \frac{1}{2} A^2 \int_{-L/2}^{L/2} \left[\sin^2\left(\frac{\pi x}{L}\right) + \cos^2\left(\frac{\pi x}{L}\right)\right] = 1$$

$$A^2 = \frac{2}{L}, \quad A = \sqrt{\frac{2}{L}}$$

Energy of lowest state

$$\psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{\pi x}{L}\right)$$

$$\frac{d^2\psi}{dx^2} = \frac{2mE}{\hbar^2}\psi$$

$$\left(\frac{\pi}{L}\right)^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{\hbar^2\pi^2}{2mL^2} = \frac{h^2}{8mL^2}$$