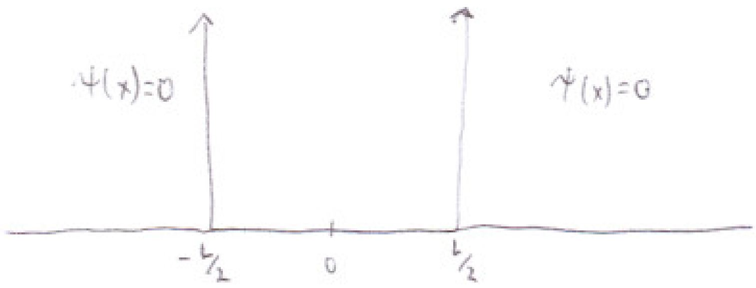
particle in a box - cont..



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\,\psi(x)$$

from boundary conditions

$$\psi\left(-\frac{L}{2}\right) = \psi\left(\frac{L}{2}\right) = 0$$

$$\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$$ - Lowest energy state

$$E = \frac{h^2}{8mL^2}$$

Higher energy states can take 2 forms

even - symmetric $\psi(x) = \psi(-x)$

odd - anti-symmetric $\psi(x) = -\psi(-x)$

Even $\psi_n(x) = A_n \cos\left(\frac{n\pi x}{L}\right)$ $n = 1, 3, 5 \ldots$

normalization condition

$$\int_{-L/2}^{L/2} dx\, A_n^2 \cos^2\left(\frac{n\pi x}{L}\right) = 1$$

gives $A_n = \sqrt{\frac{2}{L}}$ - in this case, independent of n

Odd Solutions

$$\psi_n(x) = B_n \sin\left(\frac{n\pi x}{L}\right) \qquad n = 2, 4, 6 \ldots$$

normalization condition

$$\int_{-L/2}^{L/2} dx\ B_n \sin^2\left(\frac{n\pi x}{L}\right) = 1$$

$$B_n = \sqrt{\frac{2}{L}}$$. also independent of n

so

$$\left.\begin{aligned} \psi_n(x) &= \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) \qquad n = 1, 3, 5 \ldots \\ \psi_n(x) &= \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad n = 2, 4, 6. \end{aligned}\right] \text{Energy Eigenstates}$$

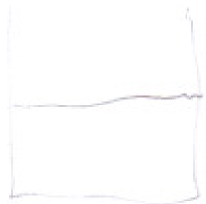
Energy

$$\frac{d^2\psi_n}{dx^2} = -\frac{2mE}{\hbar^2}\psi_n$$

$$\left(\frac{n\pi}{L}\right)^2 \psi_n = \frac{2mE_n}{\hbar^2}\psi_n$$

$$E_n = \frac{n^2 h^2}{8mL^2}$$

Compare to Uncertainty principle



from uniform distribution

$$\sigma_x = \frac{L}{\sqrt{12}}$$ - see lecture 2, example 2-3 in Rohlf

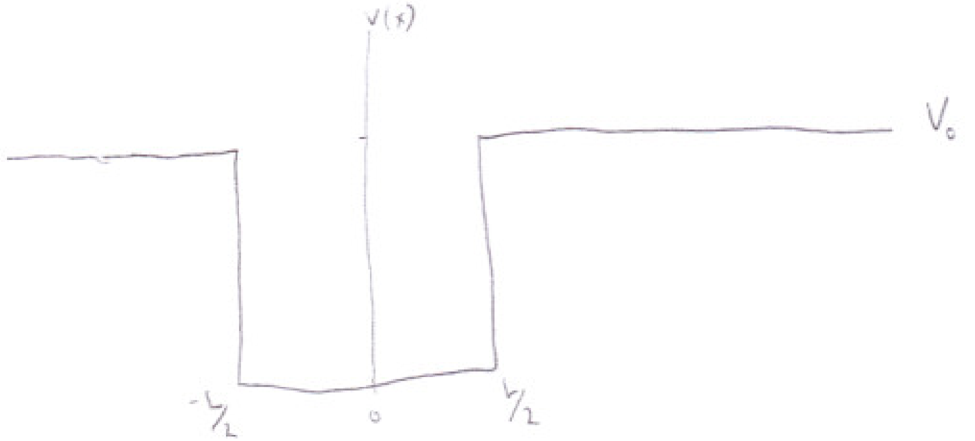
so $$\Delta p = \frac{\sqrt{12}\,\hbar}{2L}$$

$$\langle E_k \rangle = \frac{(\Delta p)^2}{2m} = \frac{3\hbar^2}{mL^2}$$

$$E_1 = \frac{\hbar^2 \pi^2}{2mL^2} \overset{?}{>} \frac{3\hbar^2}{mL^2}$$

$$\frac{\pi^2}{2} > 3$$

Finite Square potential



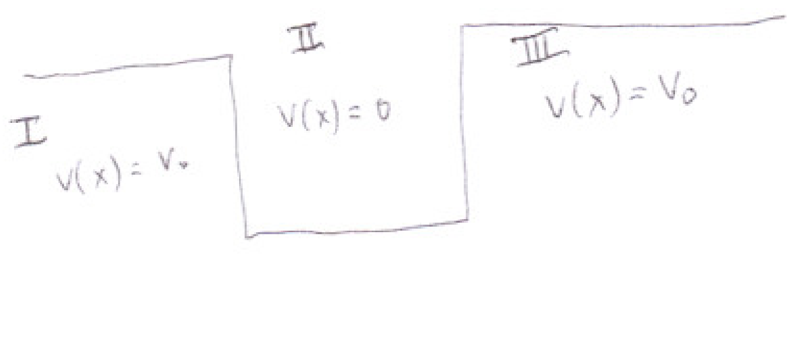
$E = \frac{p^2}{2m} + V$ Kinetic + potential energy

schrödinger eq. for particle moving in a potential

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\,\Psi(x)$$

particle is confined when $E < V_0$

Break into 3 regions, solve for each region, then connect using boundary conditions as a restraint.



In class problem # 3

for the particle in a box :

if $n = 2$, first excited state

1. draw $\psi(x)$ and $|\psi(x)|^2$

2. what is the probability of finding the particle at $x = 0$?

3. what is the average found position of the particle?