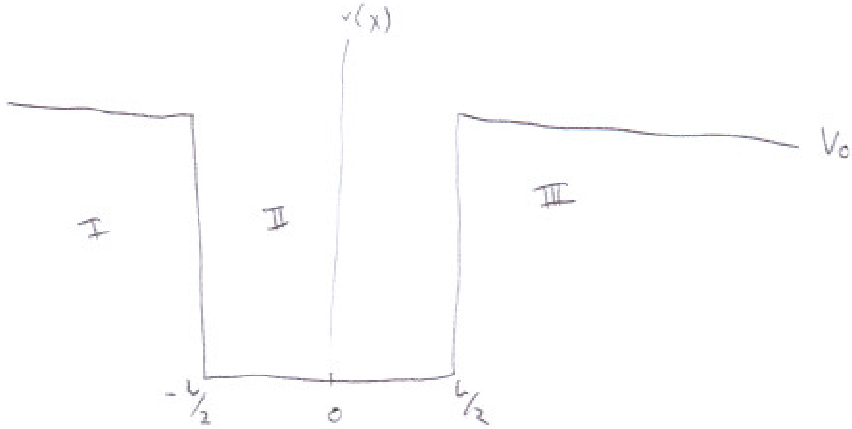
Finite Square Potential cont...



$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\,\psi(x)$$

bound states when $E < V_0$

in region I and ~~B~~ III

$$\frac{d^2\psi''}{dx^2} = \frac{2m(V_0 - E)}{\hbar^2}\psi(x)$$

$$= \beta^2\psi(x) \qquad \beta \equiv \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

region I $\psi(x) = Ce^{\beta x}$

III $\psi(x) = De^{-\beta x}$

II $\psi(x) = A\cos(kx) + B\sin(kx)$

Because of the symmetry of the potential, solutions will again be odd or even

Even Solutions

region II $\quad \psi(x) = A\cos(Kx)$

Apply boundary conditions at $+\frac{L}{2}$

Continuity of $\psi(x)$

$$A\cos\left(\frac{KL}{2}\right) = D\, e^{-\beta L/2}$$

Continuity of $\frac{d\psi}{dx}$

$$-KA\sin\left(\frac{KL}{2}\right) = -\beta D\, e^{-\beta L/2}$$

we can't get A and D, but we can find K - which tells us the allowed energies.

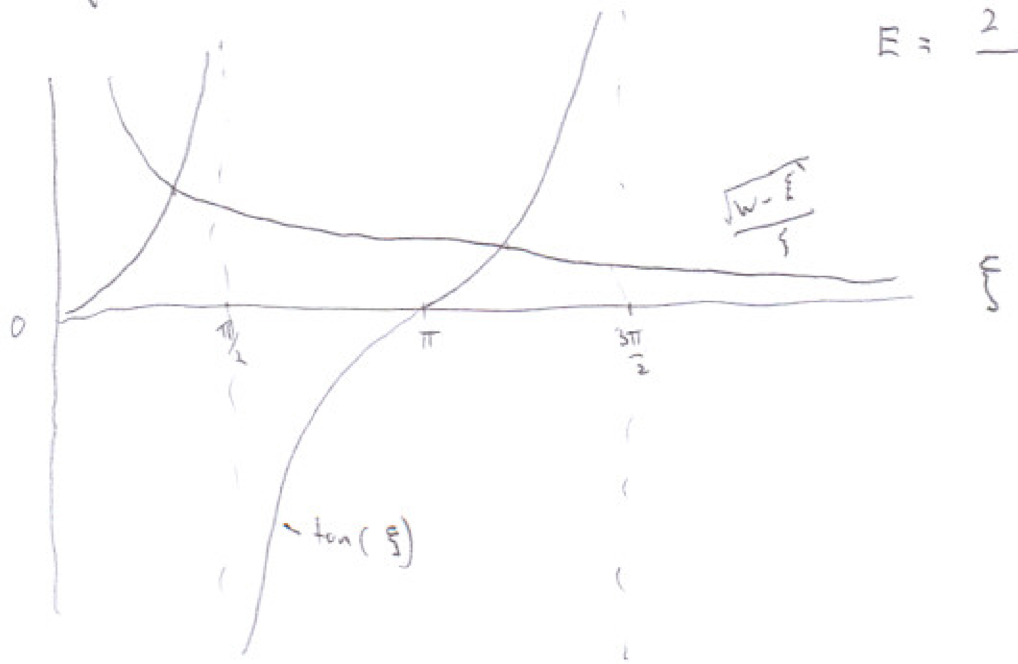
$$\tan\left(\frac{KL}{2}\right) = \frac{\beta}{K}$$

$$\tan\left(\frac{L\sqrt{2mE}}{2\hbar}\right) = \sqrt{\frac{V_0 - E}{E}}$$

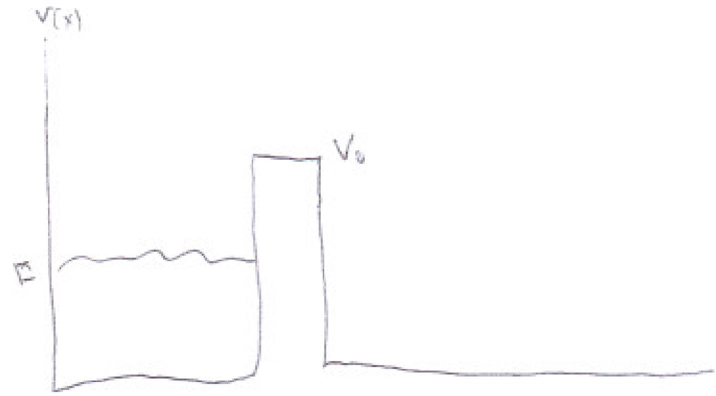
$$\xi \equiv \frac{L\sqrt{2mE}}{2\hbar}$$

$$W \equiv \frac{mV_0L^2}{2\hbar^2}$$

$$E = \frac{2\xi^2\hbar^2}{mL^2}$$



Tunneling



Classicly, a particle with $E < V_0$ could never escape

but from schrödinge eq, the particle has a chance to be found inside forbidden region