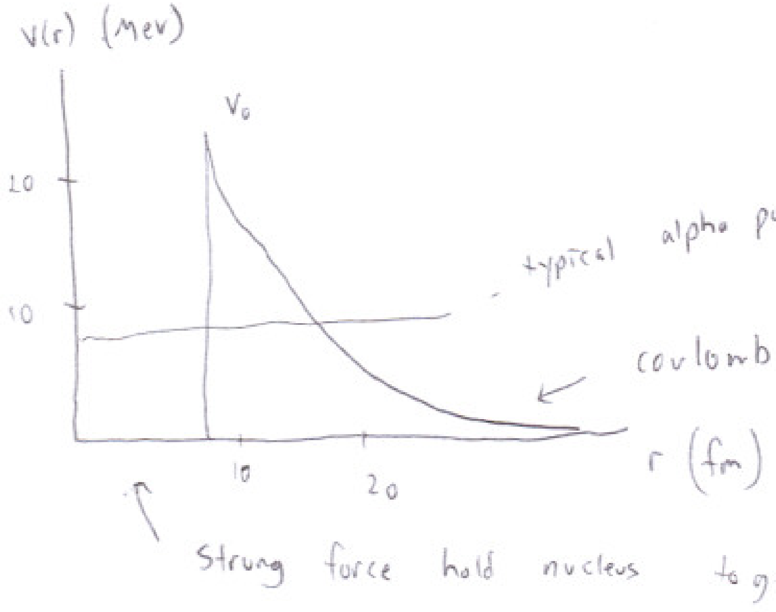
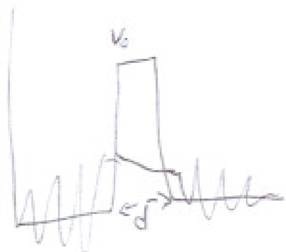
Alpha Decay



approximate barier as a square potential,



$$\beta = \sqrt{\frac{2m(V_0 - E_K)}{\hbar}}$$

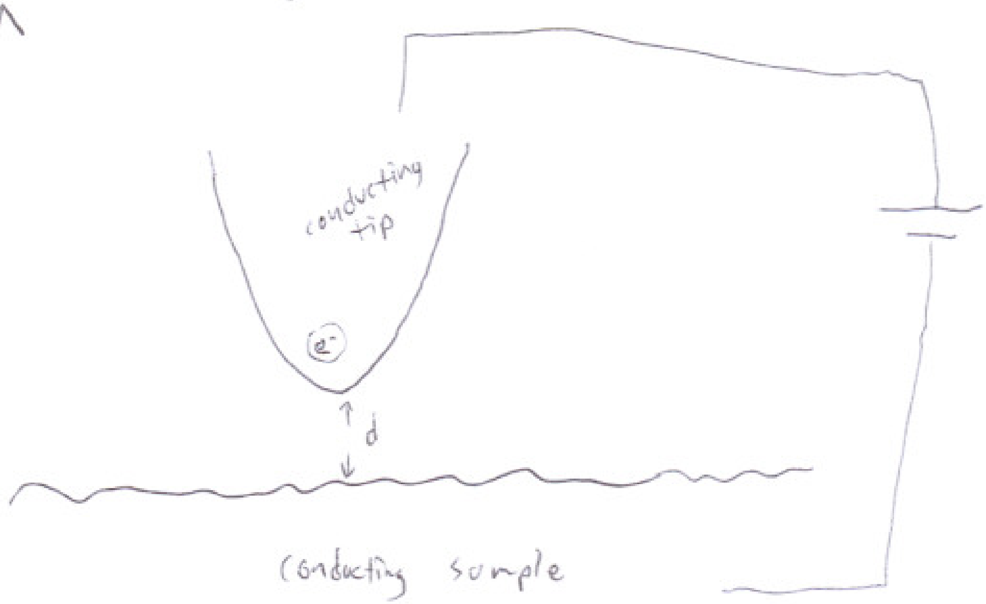
$$P \approx A^2 e^{-2\beta d}$$

d is calculated from coulomb potential

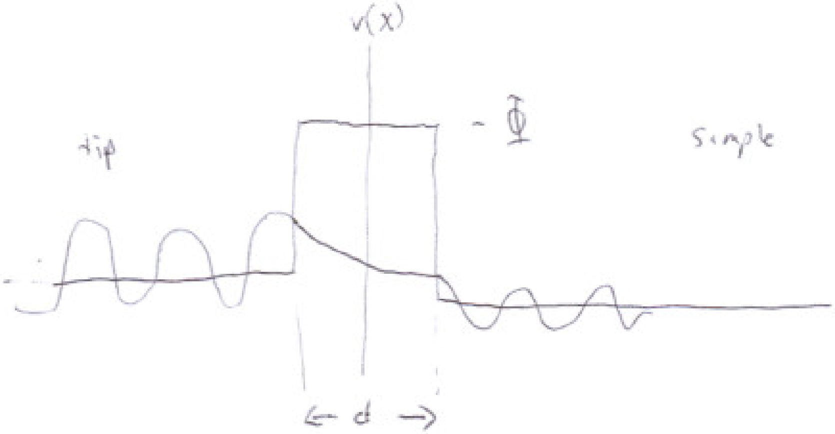
$$t_{1/2} \sim \frac{1}{P}$$

$$\ln t_{1/2} \approx C_1 - \frac{C_2 Z}{E_K}$$

Scanning Tunneling Microscope
STM



electron can't escape metal - there is a work function

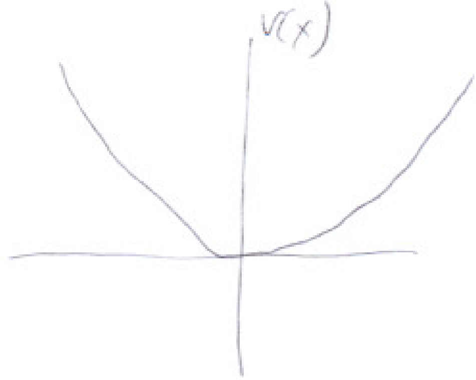


electron can tunnel through barrier

$$I \propto e^{-\frac{2d}{\hbar}\sqrt{2m\Phi}}$$

current is measured to infer d

Harmonic Oscillator



$$V(x) = \frac{1}{2} k x^2$$

for a classical mass on spring

$$x = \cos wt$$

$$mw^2 = k$$

$$V(x) = \frac{1}{2} m w^2 x^2$$ - frequency is more natural to QM than spring constant

Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2} m w^2 x^2 \psi(x) = E\psi(x)$$

The ground state:

$$\psi_1(x) = C e^{-\alpha x^2}$$ - can satisfy Schrödinger Eq.

$$\frac{d^2\psi_1(x)}{dx^2} = 4\alpha^2 x^2 \psi_1 - 2\alpha\psi_1$$

$$-\frac{\hbar^2}{2m}\left[4\alpha^2 x^2 - 2\alpha\right] + \frac{1}{2} m w^2 x^2 = E_1$$

x^2 terms must cancel since E is a constant

$$4\alpha^2 = \frac{m^2 w^2}{\hbar^2}$$

leaving $2\alpha = \frac{2mE_1}{\hbar^2}$

$$E_1 = \frac{\hbar w}{2} \qquad E_n = \left(n - \frac{1}{2}\right)\hbar w$$

Schrödinger Eq. in 3D

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V(x,y,z)\,\psi(x,y,z) = E\,\psi(x,y,z)$$

normalization condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy\,dz\,|\psi(x,y,z)|^2 = 1$$

Separation of variables

$$\psi(x,y,z) = F(x)\,G(y)\,H(z)$$

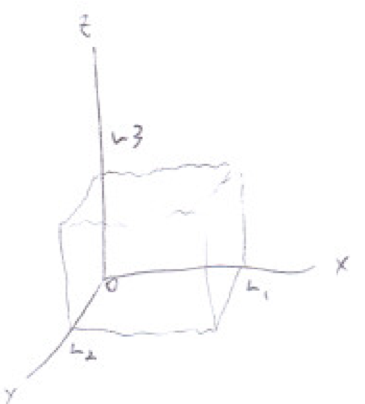
helps find solutions for 3D particle in a box

Since E is independent of x, y, or z

$$V(x,y,z) = 0 \quad \text{inside box}$$

$$-\frac{\hbar^2}{2m}\frac{d^2F}{dx^2} = C_x F$$

↑ same equation as 1D



$$\psi(x,y,z) = A \sin\left(\frac{n_1 \pi x}{L_1}\right)\sin\left(\frac{n_2 \pi y}{L_2}\right)\sin\left(\frac{n_3 \pi z}{L_3}\right)$$

$$E = \frac{\hbar^2 \pi^2}{2m}\left[\left(\frac{n_1}{L_1}\right)^2 + \left(\frac{n_2}{L_2}\right)^2 + \left(\frac{n_3}{L_3}\right)^2\right]$$

In class problem 4:

for a box of size L_1, L_2, L_3

$$L_2 = \sqrt{2}\, L_1$$

$$L_3 = \sqrt{3}\, L_1$$

a.) what is $\psi(x, y, z)$ for the first 3 lowest energies?

b.) for $\psi(x, y, z) = F(x)\, G(y)\, H(z)$

draw F, G, H for the 3 lowest energies.

c.) If $L_1 = 10$ nm, and an electron is trapped in this box, what wavelength light is emitted when going from first excited state to ground state.