Rutherford Scattering

σ - total cross section

Φ - incident flux

$$\Phi = \frac{\Delta N}{a\,\Delta t}$$

R_s - scattering rate

$$R_s = \left(\frac{\pi R^2}{a}\right)\left(\frac{\Delta N}{\Delta t}\right)$$

$$\sigma = \frac{R_s}{\Phi} = \pi R^2$$

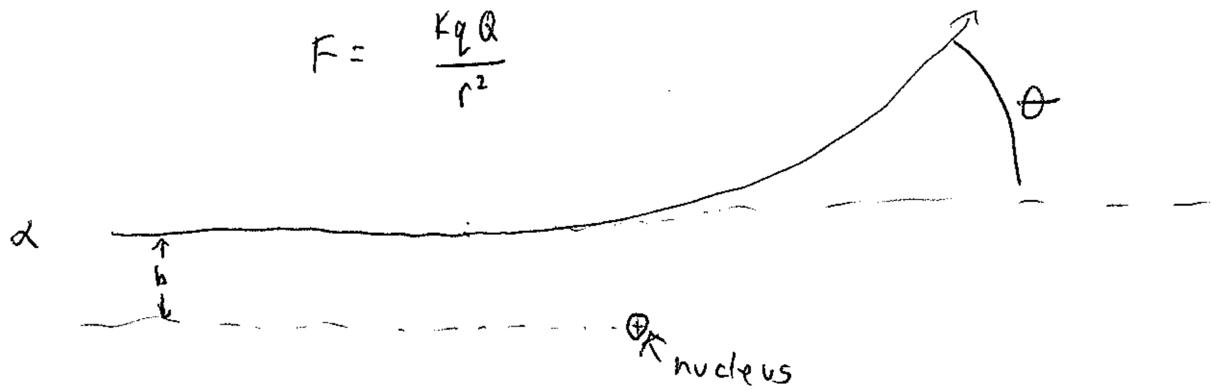
Occasional scattering at large angles sugested a nuclear model of the atom.

The angular distribution of scattered particles is the differential cross-section

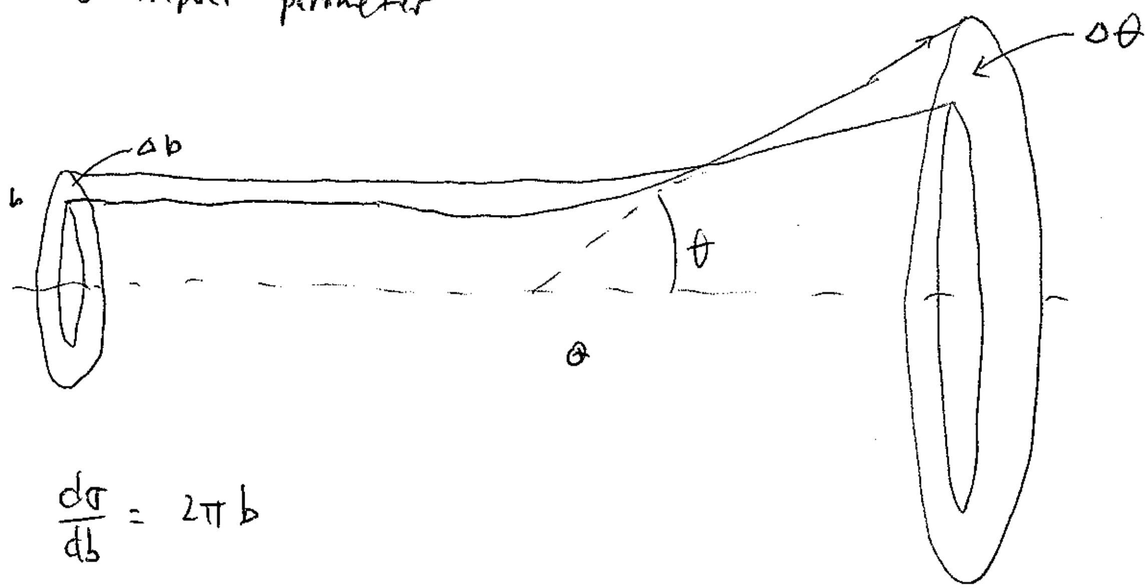
$$\frac{d\sigma}{d\cos\theta}$$

Assume scattering is from Coulomb force

$$F = \frac{kqQ}{r^2}$$



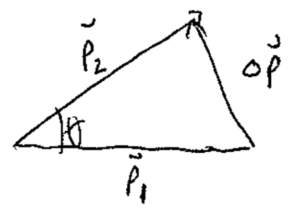
b- impact parameter



$$\frac{d\sigma}{db} = 2\pi b$$

$$\frac{d\sigma}{d\cos\theta} = 2\pi b \frac{db}{d\cos\theta}$$

now find how b depends on $\cos\theta$



$\vec{p}_1$ - before scattering

$\vec{p}_2$ - after scattering

particle doesn't change energy so $p_1 = p_2$

but

$$(\Delta p)^2 = (\vec{p}_2 - \vec{p}_1)^2 = p_2^2 - p_1^2 - 2p_1 p_2 \cos\theta$$

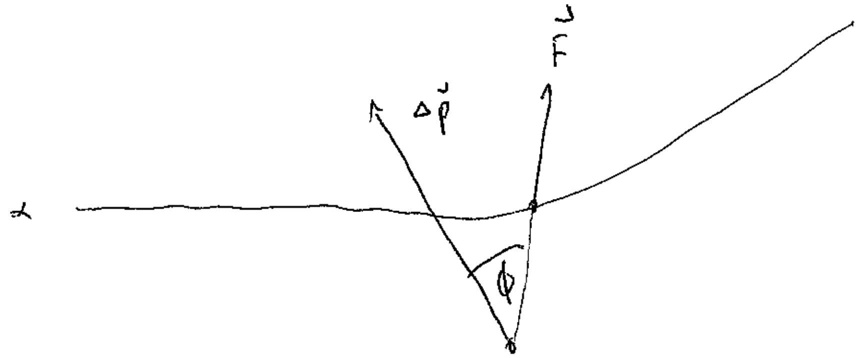
$$= 2p^2(1-\cos\theta)$$

$$\Delta p = mv\sqrt{2(1-\cos\theta)}$$

this Δp comes from the Coulomb repulsive force

$$\Delta\vec{p} = \int dt\,\vec{F}$$

we need only look in the direction of $\Delta\vec{p}$



force in the direction of $\Delta\vec{p}$ is

$$F = \frac{kq_1 q_2}{r^2} \cdot \cos\phi$$

$$\Delta p = \int_{t_1}^{t_2} dt \; F = kq_1 q_2 \int_{t_1}^{t_2} dt \; \frac{\cos\phi}{r^2}$$

ϕ, r are functions of t

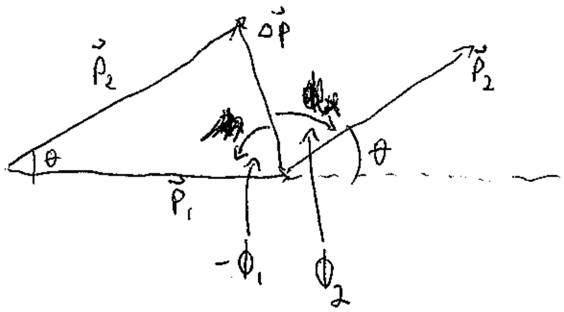
Use angular momentum conservation

$$L = mvb = m\left(r\frac{d\phi}{dt}\right)r = mr^2\frac{d\phi}{dt}$$

$$\frac{d\phi}{dt} = \frac{bv}{r^2}$$

$$\Delta p = Kq_1q_2\int_{t_1}^{t_2}\frac{\cos\phi}{r^2}\,dt \cdot \frac{d\theta}{dt}\frac{dt}{d\theta} = \frac{Kq_1q_2}{vb}\int_{\phi_1}^{\phi_2} d\phi\cos\phi$$

$$= \frac{Kq_1q_2}{vb}\left(\sin\phi_2 - \sin\phi_1\right)$$



$$\phi_2 = -\phi_1$$

$$\theta + 2\phi_2 = 180^\circ = \pi$$

$$\phi_2 = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right)$$

$$\sin(-x) = -\sin(x)$$

$$\Delta p = \frac{2Kq_1q_2}{vb}\cos\left(\frac{\theta}{2}\right) = \frac{2Kq_1q_2}{vb}\sqrt{\frac{1+\cos\theta}{2}}$$

equating 2 expressions for Δp

$$mv\sqrt{2(1-\cos\theta)} = \frac{\sqrt{2}\, K q_1 q_2}{vb}\sqrt{1+\cos\theta}$$

$$b = \frac{K q_1 q_2}{mv^2}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$\frac{d\sigma}{d\cos\theta} = 2\pi\left(\frac{K q_1 q_2}{mv^2}\right)^2 \frac{1}{(1-\cos\theta)^2}$$

$$q_1 = ze$$

$$q_2 = Ze$$

$$E = \frac{1}{2}mv^2$$

$$\alpha = \frac{2\pi K e^2}{hc} = \frac{Ke^2}{\hbar c}$$

$$z = 2$$

$$\frac{d\sigma}{d\cos\theta} = 2\pi Z^2 \alpha^2 \left(\frac{\hbar c}{E_k}\right)^2 \frac{1}{(1-\cos\theta)^2}$$