Physics 330 Exam 2
March 7, 2007
100 points total

1) For the wave function $\Psi(r,\theta,\phi)=C*r*\exp(-r/2\delta)\cos\theta$,
   a) (10 pts) Direct substitution into the Schrodinger equation reduced to

   $$-\frac{\hbar^2}{2m}\left[\frac{1}{4\delta^2}-\frac{2}{r\delta}\right]-\frac{ke^2}{r}=E$$

   Show how this determines the energy. What are E, n,l, and m_l?

   b) (10 pts) Use the normalization condition to give an expression for C. Leave in an integral form.

2) A Lithium atom (Z=3) has the electron configuration: $1s^1 2p^1 3s^1$
   a) (10 pts) Estimate the energy needed to remove all electrons.
   b) (10 pts) What are the possible configurations for lower energy states after an allowed transition?
   c) (10 pts) Which transition gives the highest energy photon? Estimate the photon's energy.

3) Consider a hydrogen atom in a n=2 state in a strong external magnetic field (B).
   a) (10 pts) Make a diagram of energy levels for all possible values of the quantum numbers m_l and m_s.
   b) (10 pts) Calculate the energy of one of the possible photon energies observable for the transition from the n=2 to n=1 state. State for which transition you are calculating.

4) For a Carbon atom (Z=6), which has two outer electrons in the 2p state:
   a) (10 pts) What are the possible values of l_{total} and s_{total}?
   b) (10 pts) Which combinations of l_{total} and s_{total} are forbidden because the total wave function isn't anti-symmetric? Justify your answer.
   c) (10 pts) What are the possible values of j_{total}?

Exam 2 Solutions

1) $\psi = C r \exp\left(\frac{-r}{2\delta}\right)\cos\theta$

a.) $$-\frac{\hbar^2}{2m}\left[\frac{1}{4\delta^2} - \frac{2}{r\delta}\right] - \frac{Ke^2}{r} = E$$

left side must be a constant, r dependence must vanish

$$\frac{\hbar^2}{mr\delta} - \frac{Ke^2}{r} = 0$$

$$\delta = \frac{\hbar^2}{mke^2}$$

then

$$-\frac{\hbar^2}{2m}\cdot\frac{1}{4\delta^2} = E$$

$$E = \frac{-\hbar^2}{8m\delta^2} = -\frac{mK^2e^4}{8\hbar^2}$$

$n=2$

$$E = -\frac{13.6\text{ eV}}{n^2} = \frac{-13.6\text{ eV}}{4} = -3.4\text{ eV}$$

if $n=2$ and ψ has θ dependence

$\ell = 1$

no ϕ dependence so

$m_\ell = 0$

b.) $\int |\psi|^2 dV = 1$ integral over all space

$$C^2\int_0^\infty dr\int_0^\pi d\theta\int_0^{2\pi} d\phi\; r^2\sin\theta\left(r^2 e^{-r/\delta}\cos^2\theta\right) = 1$$

2.) Lithium, $Z=3$ has $1s^1 2p^1 3s^1$ configuration

a.) Estimate energy to remove all electrons

$3s^1$ electron is like $n=3$ hydrogen atom

$$E_3 = \frac{+13.6\text{ eV}}{3^2} = \frac{1}{9}(13.6\text{ eV})$$

$2p^1$ see's $Z=2$, $n=2$

$$E_2 = \frac{2^2(13.6\text{ eV})}{2^2} = 13.6\text{ eV}$$

$1s^1$ see's $Z=3$, $n=1$

$$E_1 = \frac{3^2(13.6\text{ eV})}{1^2} = 9(13.6\text{ eV})$$

$$E_{total} = E_1 + E_2 + E_3 = \frac{91}{9}(13.6\text{ eV}) = 138\text{ eV}$$

b.) $\Delta l \pm 1$ gives possible states of

$1s^2 3s^1$
$1s^1 2p^2$

c.) $1s^1 2p^1 3s^1 \longrightarrow 1s^2 3s^1$ emits highest energy photon

Energy in $2p$ state $\sim -(Z-1)^2 \frac{(13.6\text{ eV})}{2^2} = -13.6\text{ eV}$

Energy in $1s$ state $\sim -(Z-1)^2 \frac{13.6\text{ eV}}{1^2} = -4 \cdot 13.6\text{ eV}$

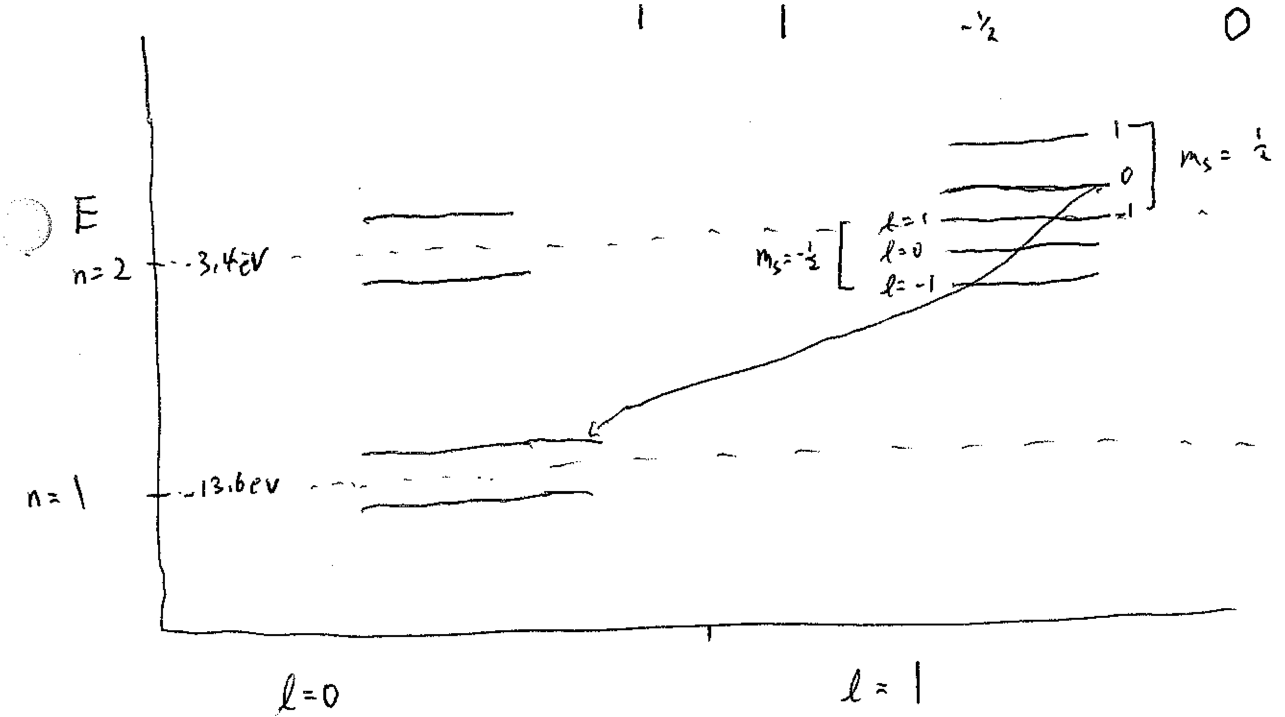
$$E \approx 3 \cdot 13.6\text{ eV}$$

3.) hydrogen, $n=2$ in strong external magnetic field B

a.) $\Delta E = \frac{e\hbar B}{2m}\left(m_\ell + 2m_s\right)$

$n=2$
$\ell=0,1$

ℓ	m_ℓ	m_s	$m_\ell+2m_s$
0	0	$\frac{1}{2}$	1
0	0	$-\frac{1}{2}$	-1
1	-1	$\frac{1}{2}$	0
1	-1	$-\frac{1}{2}$	-2
1	0	$\frac{1}{2}$	1
1	0	$-\frac{1}{2}$	-1
1	1	$\frac{1}{2}$	2
1	1	$-\frac{1}{2}$	0



b.) $n=2,\ m_s=\frac{1}{2},\ \ell=0 \longrightarrow n=1,\ m_s=\frac{1}{2},\ \ell=0$

ΔE is the same

$E = 13.6\,eV - 3.4\,eV = 10.2\ eV$

4.) Carbon, $Z=6$ has 2 outer electrons in $2p$ state

a.) only outer electrons contribute to l_{total}, S_{total}

each electron has $l=1$

$$l_{total} = |l_1 - l_2|,\ |l_1 - l_2| + 1,\ \ldots\ |l_1 + l_2|$$

$$l_{total} = 0, 1, 2$$

$s = \frac{1}{2}$ for each electron

$$S_{total} = |s_1 - s_2|,\ |s_1 - s_2| + 1,\ \ldots\ |s_1 + s_2|$$

$$S_{total} = 0, 1$$

b.) $S_{total} = 0$ - anti-symmetric

$S_{total} = 1$ - symmetric

$l_{total} = 0, 2$ - symmetric

$l_{total} = 1$ - anti-symmetric

total combined wave function must be anti-symmetric

$S_{total} = 0$, $l_{total} = 1$ - forbidden

$S_{total} = 1$, $l_{total} = 0, 2$ - forbidden

$S_{total} = 0$, $l_{total} = 0, 2$ - allowed

$S_{total} = 1$, $l_{total} = 1$ - allowed

c.) from allowed state

$S_{total} = 0$, $l_{total} = 2$

$$j = |l - s|,\ |l - s| + 1,\ \ldots\ |l + s| = 2$$

$S_{total} = 0$, $l_{total} = 0$ $\quad j = 0$

$S_{total} = 1$, $l_{total} = 1$, $\quad j = 0, 1, 2$