Physics 330 Exam 3
April 13, 2007
100 points total

1) The N_2 molecule has a vibrational frequency of about 10^{14} Hz and a bond length of 0.1 nm.

   a) (5 pts) What type of bond is this?
   b) (10 pts) What is the moment of inertia of the molecule?
   c) (10 pts) What is the rotational energy level spacing? Calculate in eV.
   d) (20 pts) The number of molecules in a given rotational state N_l is

$$N_l = A(2l+1)e^{-l(l+1)\hbar^2/2IkT}$$

   where A is some constant. At room temperature (T=300 K) what will the vibrational-rotational absorption spectrum of N_2 look like? Hint 1: The strength of peaks is proportional to the occupancy of the rotational levels before photon absorption. kT=0.026 eV at T= 300K. Hint 2: Calculate N_l at l=0,4,10. Use the approximation 1 amu = 1 GeV/c^2.

2) (25 pts) An alpha particle with energy E_k scattered with $\theta=\pi/2$. What was its angular momentum about the nucleus? Give an expression in terms of E_k , θ and Z.

3) (15 pts) A free, stationary neutron decays and you detect the resulting proton and electron. Give an expression for the kinetic energy of the electron in terms of the masses of the particles that you observed were involved in the decay. Will the measured energy always agree with you calculation? Explain your answer.

4) (10 pts) The decay process of the first four decays in the ^{238}U series are $\alpha,\beta^-,\beta^-,\alpha$. What are the daughter nuclei after each step?

Exam 3

1.)

a.) Covalent

b.) $I = \frac{m_1 m_2}{m_1 + m_2} r^2 = \left(\frac{1}{2}\right)(14)\ 1\ \text{GeV} \cdot \left(\frac{10^{-10}\ \text{m}}{3 \times 10^8\ \text{m/s}}\right)^2$

$= \frac{7}{9}\ \text{GeV} \cdot 10^{-36}\ s^2 = \frac{7}{9} \times 10^{-27}\ \text{eV} \cdot s^2$

c.) $\Delta E_r = \frac{\hbar^2}{I} = \frac{(6.6 \times 10^{-16}\ \text{eV} \cdot s)^2}{\frac{7}{9} \times 10^{-27}\ \text{eV} \cdot s^2} = \frac{9}{7} \cdot (6.6)^2 \times 10^{-5}\ \text{eV}$

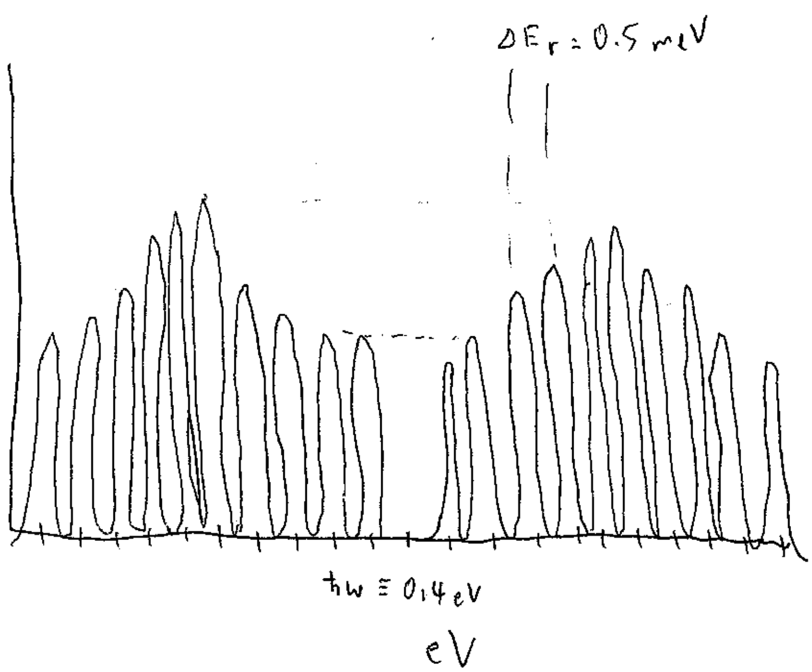
$\sim 5 \times 10^{-4}\ \text{eV} = 0.5\ \text{meV}$

d.) $\frac{\hbar^2}{2IkT} = \left(\frac{\hbar^2}{I}\right) \cdot \frac{1}{2kT} = 0.5\ \text{meV} \cdot \frac{1}{2 \cdot 26\ \text{meV}} \cong \cancel{100}\ 10^{-2}$

$l = 0 \quad \frac{N_l}{A} = e^{-10^{-2}}$

$l = 4 \quad \frac{N_l}{A} = 9\ e^{-\frac{30}{100}}$

$l = 10 \quad \frac{N_l}{A} = 21\ e^{-\frac{110}{100}}$



$\hbar\omega = \hbar \cdot 2\pi f = 6.6 \times 10^{-16}\ \text{eV} \cdot s \cdot 2\pi \cdot 10^{14}\ s^{-1}$

$\cong \cancel{3000}\ 42 \times 10^{-2}$

$\sim \cancel{3}\ .4\ \text{eV}$

2.) we used angular momentum about the nucleus in derivation of Rutherford formula.

$$L = mvb = pb = \sqrt{2mE_k}\, b$$

$$b = \frac{Kq_1q_2}{mv^2}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

for α scattering $q_1 = 2$

$q_2 = Z$

$$\theta = \frac{\pi}{2}$$

$$b = \frac{2Ke^2Z}{mv^2} = \frac{Ke^2Z}{E_K}$$

$$L = \sqrt{2mE_K} \cdot \frac{Ke^2Z}{E_K}$$

3.)

what we see is

$$n \rightarrow p + e$$

Energy conservation

$$\frac{p_n^2}{2m_n} + \frac{p_e^2}{2m_e} = \left(m_n - m_p - m_e \right) c^2$$

momentum conservation

$$p_n = p_e$$

$$\frac{p_e^2}{2m_n} + \frac{p_e^2}{2m_e} = \left(m_n - m_p - m_e \right) c^2$$

$$\frac{p_e^2}{2m_e} \left(1 + \frac{m_e}{m_n} \right) = \left(m_n - m_p - m_e \right) c^2$$

$$E_{K\text{-electron}} = \frac{(m_n - m_p - m_e) c^2}{\left(1 + \frac{m_e}{m_p} \right)}$$

This is not always the energy observed since some energy is carried away be unseen neutrino

$$n \rightarrow p + e + \bar{\nu}_e$$

4.)

$$^{238}U \longrightarrow {}^{234}Th + \alpha$$

$$^{234}Th \longrightarrow {}^{234}Pa + e^-$$

$$^{234}Pa \longrightarrow {}^{234}U + e^-$$

$$^{234}U \longrightarrow {}^{230}Th + \alpha$$