HW #2 (3)

Nelson 5.5

$D_{aorta} = 2.5 \text{ cm}$

$Q = 500 \text{ cm}^3 s^{-1}$

$\eta = 10^{-3} \text{ Pa} \cdot s$

a.) Find pressure drop per unit length along aorta
Compare drop along 10 cm to Atm. pressure (10^5 Pa)

$$Q = \frac{\pi R^4}{8 L \eta} p$$

$$\frac{dp}{dL} = \frac{8 \eta Q}{\pi R^4} = \frac{8 (10^{-3} \text{ Pa} \cdot s)(500 \times 10^{-6} m^3 s^{-1})}{\pi (1.25 \times 10^{-2} m)^4}$$

$$\frac{dP}{dL} = 52 \text{ Pa } m^{-1}$$

for $L = 10$ cm

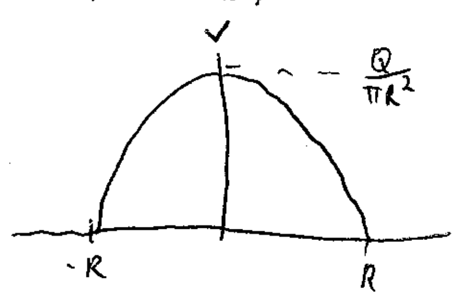
$\Delta p = 5.2$ Pa, much smaller than atmospheric pressure

b.) $P = \Delta f v = \Delta p A v = \Delta p Q = (5.2 \text{ Pa})(500 \times 10^{-6} m^{+3} s^{-1})$

$= 2.6 \text{ mJ } s^{-1} = 2.6 \text{ mW}$

Small compared to basal metabolic rate of 100 W.
Energy must be expended elsewhere

c.) $Q = \int v(r) 2\pi r dr \qquad v(r) = A(R^2 - r^2)$

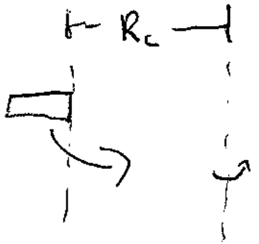


$$Q = 2\pi A \int_0^R dr (R^2 r - r^3)$$

$$Q = 2\pi A \left[\frac{R^4}{2} - \frac{R^4}{4} \right] = \frac{A \pi R^4}{2}$$

Max velocity is $A R^2 = \frac{2Q}{\pi R^2} = \frac{2 \cdot 500 \text{ cm}^3 s^{-1}}{\pi (1.25 \text{ cm})^2} = 200 \text{ cm}^{+1} s^{-1} = 2 m/s$

microcentrifuge tube $R_{tube} = 0.5$ cm, $L = 3$ cm



$R_{bac} = 1$ um

$R_{euk} = 10$ um

$\rho_{cell} = 1.05\ \rho_{H_2O}$

$R_c = 10$ cm

centrifugal "force" is $f = m_{net}\, a$

$$a = w^2 R_c$$

$$V_{drift} = \frac{f}{\zeta}, \quad \zeta = 6\pi\eta R$$

$$T = \frac{L}{V_{drift}} = \frac{L\zeta}{f} = \frac{6L\pi\eta R}{m_{net}\, w^2 R_c}$$

$$w^2 = \frac{6L\pi\eta R}{T m_{net} R_c} = \frac{6L\pi\eta R}{T(.05\,\rho_{H_2O})\frac{4}{3}\pi R^3}$$

$$w^2 = \frac{9\eta L}{2TR_c R^2(.05\,\rho_{H_2O})}$$

for R_{bac}
$$w^2 = \frac{9\left(10^{-3}\,\mathrm{Kg\,m^{-1}s^{-1}}\right)(.03\,\mathrm{m})}{2(300\,\mathrm{s})(.1\,\mathrm{m})(10^{-6}\,\mathrm{m})^2(.05 \times 1000\,\mathrm{Kg\,m^{-3}})}$$

$w^2 = 9 \times 10^4\ \mathrm{rad^2/s^2}$

$w = 300\ \mathrm{s^{-1}}$

$f = 48\ \mathrm{s^{-1}} \sim 2900$ rpm

$a = w^2 R_c = 9000\ \mathrm{m\,s^{-2}} \cong 900g$

for R_{euk} $w^2 = 900\ \mathrm{s^{-2}}$

$w = 30\ \mathrm{s^{-1}}$

$f = 4.7\ \mathrm{s^{-1}} = 290$ rpm

$a = 90g$

HW #2

(5)

from Ehrensperger:

$$P_{IN} = \frac{\alpha \exp\left(-U_0/k_BT\right)}{1-\alpha + \alpha \exp\left(-U_0/k_BT\right)}$$

$U=0$ out side clusters

$U=U_0<0$ inside clusters

α = fraction of surface occupied by clusters
can be measured experimentally

P_{IN} : probability of finding a receptor in a cluster

from Nelson 6.6.4

in a 2 state system $\frac{P_I}{P_{II}} = \frac{Z_I}{Z_{II}}$.

and in general: $P_I = Z^{-1} \sum_{j=1}^{N} e^{-E_j/k_BT}$

where $z = \sum_j e^{-E_j/k_BT}$ sum over all possible states

If number of states is proportional to area,
the 2 states are (1) in a cluster, or (2) out of a cluster

then

$$\boxed{P_{IN} = \frac{\alpha \exp\left[-U_0/k_BT\right]}{(1-\alpha)\exp\left[-0/k_BT\right] + \alpha \exp\left[-U_0/k_BT\right]}}$$

now its clear the denominator is the partition function.

HW #2

(6) Derive the Hagen-Poiseuille relation for flow in a pipe

Navier - Stokes

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = f - \frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{v}$$

$$\nu \equiv \frac{\eta}{\rho}$$

for steady, irrotational flow this becomes

$$\frac{1}{\rho}\nabla p = \nu \nabla^2 \vec{v}$$

or

$$\nabla p = \eta \nabla^2 \vec{v}$$

fluid motion and pressure drop only along z

$$\frac{\partial p}{\partial z} = \eta \nabla^2 v_z(r)$$

There is a uniform pressure gradient so

$$\nabla^2 v_z(r) = \frac{-\Delta p}{\eta L}$$

$$\frac{1}{r}\frac{d}{dr} r \frac{dv_z(r)}{dr} = -\frac{\Delta p}{\eta L}$$

Integrate twice

$$v_z(r) = \frac{-\Delta p r^2}{4\eta L} + c_1 \ln r + C_2$$

$v_z(R) = 0$ viscous boundary condition

$$v_z(r) = \frac{\Delta p}{4\eta L}(R^2 - r^2)$$

$$Q = \rho \int r \, dr \, d\phi \; v_z(r) = \frac{\pi R^4 \Delta p}{8 L \eta}\rho$$