HW #3

7.3

artificial blood cell

$R = 10\,\mu m$

filled with hemoglobin

Cells burst in pure water, stable with 1mM NaCl

a.) If osmoticaly balanced at 1 mM salt,
2 mM will cause dehydration

b.) Salt disassociates in water, each Na and Cl
contributes to osmotic pressure.
we need 2 mM glucose.

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a.) Blood plasma protein osmotic pressure is $P = 28$ mm Hg

$$1 \text{ atm} = 760 \text{ mm Hg} = 10^5 \text{ Pa}$$

In the dilute limit

$$PV = NKT$$

$$\frac{N}{V} = \frac{P}{KT}$$

express concentration as moles L^{-1}

$$c = \frac{P}{KT \cdot N_{mole} \left(1000 \frac{L}{m3}\right)} = \frac{(28 \text{ mm Hg})\left(\frac{10^5 \text{ Pa}}{760 \text{ mm Hg}}\right)}{(4.1 \times 10^{-21} \text{ J})(6.03 \times 10^{23})\left(1000 \frac{L}{m3}\right)}$$

$$c = 1.5 \text{ mM} = 1.5 \times 10^{-3} \text{ moles } L^{-1}$$

molar mass $M = \dfrac{60 \frac{g}{L}}{1.5 \times 10^{-3} \text{ moles}/L} = 4 \times 10^5 \dfrac{g}{\text{mole}} = 40$ kDa

"Dalton"

7.4

b.) filtration coefficient: $L_p = 7 \times 10^{-6}$ cm s^{-1} atm^{-1}

$$j_v = -L_p\left(\Delta p - (\Delta c) K_B T\right)$$

assume no applied pressure

$$j_v = +L_p (\Delta c) K_B T$$

total accumulation is

$$Q_t = A j_v t = A t L_p (\Delta c) K_B T$$

using values in part a.)

$$(\Delta c) K_B T = (0.1)(28 \text{ mm Hg})\left(\frac{1 \text{ atm}}{760 \text{ mm Hg}}\right)$$

$$Q_t = (250 \text{ m}^2)(86400 \text{ s})(7 \times 10^{-6} \text{ cm s}^{-1} \text{ atm}^{-1}) \times (0.1)(28 \text{ mm Hg})\left(\frac{1 \text{ atm}}{760 \text{ mm Hg}}\right)$$

$$= 0.56 \text{ m}^2 \text{ cm} = 5.6 \text{ L} \quad \text{in one day}$$

5.6 L/day

reduction in food ⟶ reduction in blood plasma protein

fluid (H_2O) is lost from capillaries to interstitial space leading to swelling

Hw #3

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pH of H_2O at $0°C$ is 7.5

at $40°C$ is 6.8

$pH = -\log_{10}[H+]$ $[H+]$ is concentration in $\frac{\text{moles}}{L}$

At higher temp there is more dissociation, therefor more free H+, this means lower pH

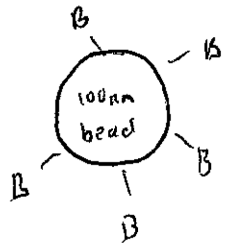
we can see this from

$$K_{eq} = e^{-\Delta G°/k_BT} = \frac{[H+][OH]}{[H_2O]}$$

as $T \to 0$ $[H+] \to 0$

HW #3

3.)



B binds H

$$BH \rightleftharpoons B + H$$

$$K_d = \frac{[H][B]}{[BH]}$$

at pH = 6 50% of B have bound on H

$$[B] = [BH]$$

$$K_d = 10^{-6}$$

define occupation ratio $\eta \equiv \dfrac{[BH]}{[BH]+[B]} = \dfrac{1}{1 + \frac{[B]}{[BH]}}$

$$\eta = \frac{1}{1 + \frac{K_d}{[H]}}$$

$$pH \equiv -\log_{10}[H]$$

$$\eta = \frac{1}{1 + 10^{pH-6}}$$

$$\frac{d\eta}{d(pH)} = \eta^2 \log(10)\, 10^{pH-6}$$

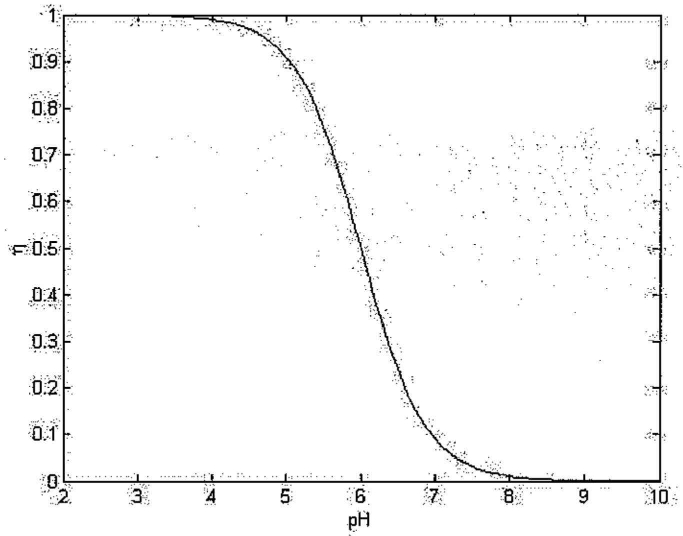
see Matlab code and plots ...

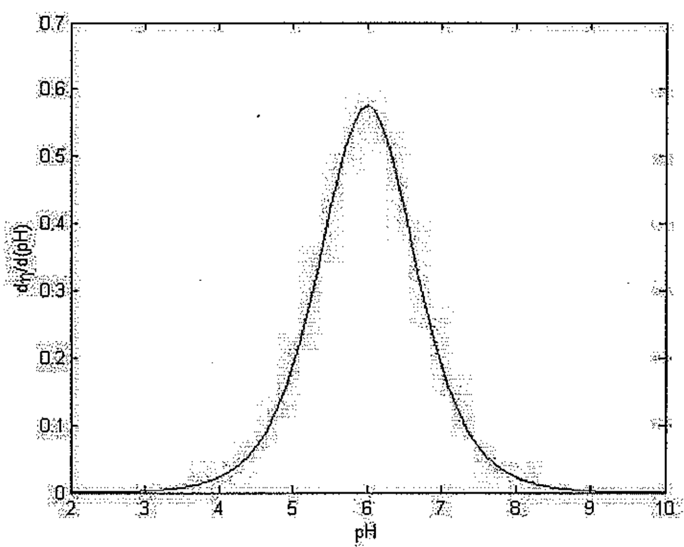
```
pH=(2:.1:10) %make a x-axis
n=1./(1+10.^(pH-6));
dndpH=n.*n.*log(10).*10.^(pH-6);

%plot these:

figure
plot(pH,n,'b-')
xlabel('pH')
ylabel('\eta')

figure
plot(pH,dndpH,'r-')
xlabel('pH')
ylabel('d\eta/d(pH)')
```

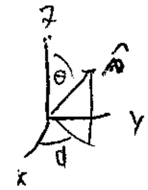




HW#3

6.9

System has energy as continous variable



$$E = -a\hat{n}\cdot\hat{z} = -a\cos\theta$$

a.) $$P(\theta,\phi)\,d\theta\,d\phi = Z^{-1}\exp\left[a\cos\theta/k_BT\right]$$

$$\beta \equiv \frac{1}{k_BT}$$

b.) $$Z = \int_0^{\pi}\int_0^{2\pi} d\theta\sin\theta\,d\phi\,\exp\left[a\beta\cos\theta\right]$$

$$x = \cos\theta$$
$$dx = -\sin\theta\,d\theta$$

$$Z = 2\pi\int_{-1}^{1} -dx\,\exp\left[a\beta x\right] = -\frac{2\pi}{a\beta}\left[e^{a\beta} - e^{-a\beta}\right] = \frac{-4\pi}{a\beta}\sinh(a\beta)$$

$$F = -k_BT\ln Z = -k_BT\left[\ln\frac{a\beta}{4\pi} + \ln\sinh(a\beta)\right]$$

$$\langle\hat{n}_z\rangle = \langle\cos\theta\rangle = Z^{-1}\int_0^{\pi}\int_0^{2\pi} d\theta\sin\theta\,d\phi\,\cos\theta\,\exp\left[a\cos\theta/k_BT\right]$$

$$= \frac{2\pi}{Z}\int_{-1}^{1} -x\,dx\,e^{a\beta x} = \left(\frac{2\pi}{Z}\right)\frac{1}{(a\beta)^2}\left[e^{a\beta x}(1-a\beta x)\right]_{-1}^{1}$$

$$= \frac{4\pi}{Z(a\beta)^2}\left(\sinh a\beta - a\beta\cosh a\beta\right)$$

$$\langle\hat{n}_z\rangle = \coth a\beta - \frac{1}{a\beta}$$