Physics 480: Homework #2
Due October 11, 2010
10 points

Many proteins are confined to the cytoplasmic membrane due to one or more hydrophobic regions in the protein. A particular protein is present throughout most of the membrane at a concentration of 50 proteins per μm^2 and is found to have a diffusion constant of D=0.1 μm^2/s. A local change in the composition of the membrane effectively gives a change in the proteins energy such that $U = U^0 - B\exp[-\frac{x^2+y^2}{2\sigma^2}]$, where $\sigma = 0.1$ μm.

1. (4 points) Use the principle that chemical potential is constant across systems that are in contact and in equilibrium to find the expression for the expected concentration as a function of position. Assume the concentration is maintained at 50 μm^{-2} far from $x, y = 0, 0$. Calculate (numerically) the expected number of proteins within 2σ of the origin. Make calculations for $B = K_BT$, $2K_BT$, and $5K_BT$.

2. (4 points) Compare the above results to that from a simulation. Create a simulation of particles diffusing in 2D and allow the system to come to equilibrium by updating positions after many time steps. Compare the number of particles within 2σ of the origin to that calculated above. Provide your m-file with solutions. Hints: (1) $F = -\nabla U = v\xi$; (2) Take time steps such that $< x^2 > \ll \sigma^2$; (3) Simulate a region of size $L \gg \sigma$ so concentration is negligibly affected far from the origin.

3. (2 points) Use Nelson Eq. 6.6 and surrounding discussion to show that in 2D the chemical potential can still be written as $\mu = \mu^0 + K_BT \log c$ where c is the 2D concentration.