Home work # 2

(1.) $$U = U^0 - B\exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]$$

$$\mu = \mu_0 + K_BT\ln c + V(\vec{r})$$

$$\mu(x,y) = \mu_0 + K_BT\ln C + U^0 - B\exp\left[-\frac{x^2+y^2}{2\sigma^2}\right]$$

At Eq. $\mu(x,y)$ is constant across the membrane

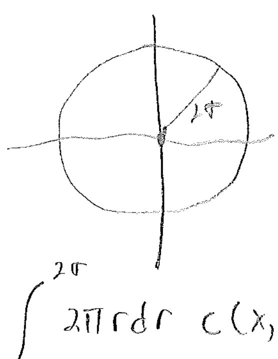
$$K_BT\ln c(x,y) = B\exp\left[-\frac{x^2+y^2}{2\sigma^2}\right] + \text{const}$$

$$c(x,y) = \text{const} \times \exp\left[\frac{B}{KT} e^{-\frac{x^2+y^2}{2\sigma^2}}\right]$$

$$c(\infty,\infty) = C_\infty = 50\ \mu m^{-2}$$

$$c(x,y) = C_\infty \exp\left[\frac{B}{KT} e^{-\frac{x^2+y^2}{2\sigma^2}}\right]$$

Expected number within 2σ



for $C_\infty = 50\ \mu m^{-2}$

$\sigma = 0.1\ \mu m$

$$N = \int_0^{2\sigma} 2\pi r\,dr\; c(x,y)$$

$B = K_BT$, $N = 10$ $B = 2K_BT$, $N = 17$ $B = 5K_BT$, $N = 123$

2.

See 'Diffusion In Potential Well, m'

3.

$$\frac{S}{K_B} = \ln\left[\frac{2\pi^{\eta N/2}}{(\eta N/2 - 1)!} (2mE)^{\eta N/2} V^N \frac{1}{N!} (2\pi\hbar)^{-\eta N} \frac{1}{2}\right]$$

V is the n-dimensional volume

for large N, use stirling approximation

$$\ln x! \approx x\ln x - x \approx x \ln x = \ln x^x$$

and $\frac{\eta N}{2} - 1 \rightarrow \frac{\eta N}{2}$

then

$$\frac{S}{K_B} = \ln\left[\left(2\pi^{\eta N/2}\right)\left(\frac{\eta N}{2}\right)^{-\eta N/2} (2mE)^{\eta N/2} V^N N^{-N} (2\pi\hbar)^{-\eta N} \frac{1}{2}\right]$$

$$= \ln\left[\left(\frac{E}{N}\right)^{\eta N/2} \left(\frac{V}{N}\right)^N\right] + \ln\left[\left(2\pi^{\eta N/2}\right)\left(\frac{\eta}{2}\right)^{-\eta N/2} (2m)^{\eta N/2} (2\pi\hbar)^{-\eta N} \frac{1}{2}\right]$$

$$= N\ln\left[\left(\frac{E}{N}\right)^{\eta/2}\left(\frac{V}{N}\right)\right] + \frac{\eta N}{2}\ln\left[\frac{8\pi\eta m}{h^2}\right] + \ln\frac{1}{2}$$

$$\mu = -T\left(\frac{dS}{dN}\right)_{E,V}$$

$$\frac{1}{K_B}\frac{dS}{dN} = \ln\left[\left(\frac{E}{N}\right)^{\eta/2}\frac{V}{N}\right] + \frac{N}{\left(\frac{E}{N}\right)^{\eta/2}\left(\frac{V}{N}\right)}\left(-\frac{\eta}{2}\frac{E^{\eta/2}}{N^{(\eta/2+1)}}\frac{V}{N} - \left(\frac{E}{N}\right)^{\eta/2}\frac{V}{N^2}\right)$$

$$+ \frac{\eta}{2}\ln\left[\frac{8\pi\eta m}{h^2}\right]$$

$$\frac{1}{K_B}\frac{dS}{dN} = \ln\left[\left(\frac{E}{N}\right)^{n/2}\left(\frac{V}{N}\right)\right] - \left(\frac{n}{2}+1\right) + \frac{n}{2}\ln\left(\frac{8\pi n m}{h^2}\right)$$

$$E = \frac{nN}{2} K_B T \qquad \text{(ideal gas)}$$

$$\mu = -K_B T \ln\left(\frac{V}{N}\right) + \text{const}(T)$$

$$\mu = \mu_0 + K_B T \ln c$$ where c is n-dimensioned concentration.