

Physics 480 and 581: Homework #3  
Due November 9, 2011

1. (10 points)

Dilute solutions can often be treated as an ideal gas. Use the Sackur-Tetrode equation [http://en.wikipedia.org/wiki/Sackur-Tetrode\\_equation](http://en.wikipedia.org/wiki/Sackur-Tetrode_equation) and the relation  $\mu = -T\left(\frac{dS}{dN}\right)_{E,V}$  to show that for an ideal gas,  $\mu = \mu^0 + RT \ln c$  where  $c$  is the concentration with respect to a reference concentration. Does the reference concentration affect  $\mu^0$ ?

2. (10 points)

A cylindrical transmembrane protein ( $R=1.0$  nm) is found to diffuse in a lipid membrane with  $D = 1.0 \mu\text{m}^2\text{s}^{-1}$ . Use the Saffman-Delbrück theory to find the diffusion constant of a cluster of these proteins as a function of the number of proteins. Plot your results from  $N = 1$  to  $N = 100$ . Clearly state your assumptions.

3. (581 only: 10 points)

Show that for a particle undergoing Brownian motion and trapped in a 1D box of length  $L$

$$\langle x^2 \rangle (t) = \frac{L^2}{6} - \frac{16L^2}{\pi^4} \sum_{n=1(\text{odd})}^{\infty} \frac{1}{n^4} \exp\left(-\frac{1}{2}\left(\frac{n\pi\sigma}{L}\right)^2 t\right)$$

where  $\sigma^2 = 2D$ . Hint: See Kusumi, Biophysical Journal, vol 65, 2021-2040, 1993.

What is  $\langle r^2 \rangle$  for a particle trapped in a rectangular volume of size  $L_1, L_2, L_3$ ?