(1.)

$$S = kN \ln\left[\left(\frac{V}{N}\right)\left(\frac{U}{N}\right)^{3/2}\right] + \frac{3}{2}kN\left(\frac{5}{3} + \ln\frac{4\pi m}{3h^2}\right)$$

$$\mu = -T\left(\frac{dS}{dN}\right)_{U,V}$$

$$\frac{dS}{dN} = k\ln\left[\left(\frac{V}{N}\right)\left(\frac{U}{N}\right)^{3/2}\right] + \frac{kN}{\left(\frac{V}{N}\right)}\left(-\frac{V}{N^2}\right) + \frac{3kN}{2}\frac{kN}{\left(\frac{U}{N}\right)}\left(-\frac{U}{N^2}\right)$$

$$+ \frac{3}{2}k\left(5/3 + \ln\frac{4\pi m}{3h^2}\right)$$

$$= k\ln\left[\left(\frac{V}{N}\right)\left(\frac{U}{N}\right)^{3/2}\right] - k - \frac{3}{2}k + \frac{15}{6}k + k\ln\frac{4\pi m}{3h^2}$$

$$= k\ln\left[\left(\frac{V}{N}\right)\left(\frac{U}{N}\right)^{3/2}\right] + A$$

$$A \equiv \frac{5}{6}k + k\ln\frac{4\pi m}{3h^2} \qquad \frac{U}{N} = \frac{3}{2}kT$$

for ideal gas.

$$c = N/V$$

$$\mu = -kT\ln\left(\frac{1}{c}\right) - \frac{3}{2}kT\ln\left(\frac{3}{2}kT\right) - AT$$

$$\mu = kT\ln\left(\frac{c}{c_0}\right) + \mu_0(T) \qquad \mu_0(T) = -kT\ln c_0 - \frac{3}{2}kT\ln\left(\frac{3}{2}kT\right) - AT$$

using moles instead of particles gives us RT

reference concentration affects $\mu_0(T)$

2.

$$D = \frac{K_B T}{4\pi \eta_m h}\left[\ln\left(\frac{h\eta_m}{a\eta_w}\right) - \gamma\right] \qquad \gamma \sim 0.577$$

assume η_w is viscosity of water

$$\eta_w \approx 10^{-3} \ Pa.s$$

when $a = 1.0 \ nm$, $D = 1 \mu m^2/s$.

solve for η_m

$$\eta_m = \frac{K_B T}{4\pi D h}\left[\ln\left(\frac{h\eta_m}{a\eta_w}\right) - \gamma\right]$$

← this is transcendental equation.

see HW3-2.m

For N proteins in a cluster, from conservation of area,

$$r_N^2 \approx N r_1^2$$

$$r_N = \sqrt{N}\, r_1$$

3.

$$\frac{\partial P(x,t)}{\partial t} = D \nabla^2 P(x,t)$$

for a closed 1D box of length L,

$$\left.\frac{\partial P}{\partial x}\right|_{x=0} = \left.\frac{\partial P}{\partial x}\right|_{x=L} = 0$$ ← no flux through boundary

write

$$P(x,t) = X(x)\,T(t)$$

$$X(x)\frac{dT(t)}{dt} = D\,T(t)\frac{d^2X(x)}{dx^2}$$

can show that λ must be positive

$$\frac{1}{D}\frac{T'}{T} = \frac{X''}{X} = -\alpha$$

zero from B.C.

$$X(x) = \cancel{A}\sin(\sqrt{\lambda}\,x) + B\cos(\sqrt{\lambda}\,x)$$ and $\sqrt{\lambda} = \frac{n\pi}{L}$

$$T(t) = C e^{-\lambda D t} = C e^{-(n\pi/L)^2 D t}$$

$$P(x,t) = \frac{P_0}{2} + \sum_{n=0}^{\infty} D_n \cos\left(\frac{n\pi x}{L}\right) e^{-(n\pi/L)^2 D t}$$

$$D_n = \frac{2}{L}\int_0^L f(x,0)\cos\left(\frac{n\pi x}{L}\right)dx$$

for a particle initially at x,

$$f(x,0) = \delta(x)$$

$$D_n = \frac{2}{L}\int_0^L \delta(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)$$

to find $\langle x^2\rangle$, average over all possible start and end positions.

$$P(y,t\,|\,x,0) = \frac{1}{L} + \frac{2}{L}\sum_{n=1}^{\infty}\cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)e^{-(n\pi/L)^2 Dt}$$

$$P(x,0) = \frac{1}{L}$$

$$\langle x^2\rangle = \int_0^L\int_0^L (y-x)^2\, P(y,t\,|\,x,0)\,P(x,0)\,dx\,dy$$

$$= \frac{1}{L}\int_0^L\int_0^L (y-x)^2\left[\frac{1}{L} + \frac{2}{L}\sum\cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)e^{-\left(\frac{n\pi}{L}\right)^2 Dt}\right]dx\,dy$$

$$\int_0^L\int_0^L (y^2-2xy+x^2)\,dx\,dy = L^4/6$$

$$\int_0^L\int_0^L \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)x^2\,dx\,dy = 0$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right)x\,dx = L^2\frac{(\cos(\pi n)-1)}{\pi^2 n^2} = -\frac{2L^2}{\pi^2 n^2} \quad n \text{ odd}$$

$$0 \quad n \text{ even}$$

$$\langle x^2 \rangle = \frac{L^2}{6} - \frac{16 L^2}{\pi^4} \sum \frac{1}{n^4} e^{-\left(\frac{n\pi}{L}\right)^2 D t}$$

for a rectangular volume of size

$$L_1, L_2, L_3$$

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

$$= \sum_{i=1}^{3} \left[\frac{L_i^2}{6} - \frac{16 L_i^2}{\pi^4} \sum \frac{1}{n^4} e^{-(n\pi/L_i)^2 D t} \right]$$