Physics 493L: Homework #1
Due March 2, 2010
10 points

Show in the case where a model is linear with respect to a set of parameters and the data is normally distributed that the maximum likelihood estimation of the parameters is equal to the expression

$$\hat{\theta} = (X'X)^{-1}X'y$$

where $\theta$ is a $P \times 1$ vector of parameters, $X$ is a $N \times P$ 'design matrix' ($X'$ means the transpose of $X$) and $y$ is the observed data set. For example, in the case of a fit to a line $y = Ax + B$, and $\theta = [A\ B]'$. For data sampled at points $x_n$, the elements of $X$ would be $X_{n,1} = 1$ and $X_{n,2} = x_n$.

For the following, write your own MATLAB script and functions. Create a data set using the model $y = Ax + B$. Corrupt the result with normally distributed noise (see 'randn'). Use the above expression to find $A$ and $B$. Also find $A$ and $B$ by minimizing the negative of the log likelihood function using 'fminsearch' (see example posted on our class page.) Show that $A$ and $B$ found using the two methods are identical. Plot your results. Remember to use axis labels and a legend.

Turn in your derivation and the MATLAB m-files (via e-mail) that I can use to reproduce your results.