Physics 493L: Homework #2
Due April 15, 2010
10 points

The underlying model of an exponential decay signal in the presence of a constant background is given by:

$$u(t) = \theta_A e^{-t/\theta_\tau} + \theta_B$$

In this homework you will make a maximum likelihood estimate of the parameters θ_A, θ_τ and θ_B, show that the MLE is an efficient estimator, and give the precision of the estimates of each of the parameters.

Take $\theta_A = 100, \theta_\tau = 2$ μs and $\theta_B = 10$. Assume you have time bins of width $\Delta t = 10$ ns and that you collect data up to $t = 10$ μs.

1. **Make a Maximum Likelihood Estimate of the parameters.** Generate the model using the values above. Make a noisy 'data set' by selecting a random a value from a Poisson distribution, which has the expected value given by the model. You can use the function linked on our website 'GenPoisson' to do this. Show the derivation of the expression for the LogLikelihood. Modify your code from HW1 so that it now minimizes the negative LogLikelihood. Again, use 'fminsearch' and your custom function to find the MLE of the parameters.

2. **Show that the MLE is an efficient estimator.** Derive the Cramer-Rao Lower Bound for the three parameters. Hint: use the approach

   $$I_{ij}(\theta) = -E\left[\frac{\partial^2 \ln(L(\vec{x}|\theta))}{\partial\theta_i \partial\theta_j}\right]$$

   and remember that the expected value for the data is the model.

   Now calculate many 'observations' by making new noisy data sets and then use your code from above to estimate θ_A, θ_τ and θ_B. Is the variance of the estimates similar to your calculated CRLB?

3. **Give the precision of the estimates.** If the variance of the estimates approach the CRLB, you can use the square root of the CRLB to report the uncertainty in a single measurement. What is this for each of the parameters?

Turn in your MATLAB code by e-mail and your written derivations on paper or by e-mail.