LAB 3: Speed Measurement by Optical Techniques

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Purpose:

Familiarize the student with the Doppler effect as used for speed measurement, spatial filtering, spectrum analyzers, and interferometric techniques.

Reading Assignment

As referenced in text, and Chapter 4 of *Building Scientific Apparatus*, 3rd edition, by John Moore et al. (Perseus Books, Cambridge MA, 2003).

Introduction

The most obvious optical method for measuring the speed of a moving object is to use the Doppler effect: light reflected from the moving object will be shifted in frequency in proportion to the line of sight velocity of the object. While we are familiar with examples in the radio domain (such as police traffic radar and Doppler weather radar) and with acoustic waves (the apparent change in pitch of a passing siren), demonstrating the Doppler effect optically is not a trivial matter. The primary reasons for this are:

1. Optical frequencies are much too high (~ 10^{14} Hz) to be measured directly, and
2. The shift corresponding to typical speeds is on the order of 0.1 - 10 MHz, much less than the line width of a typical He-Ne laser.

It is nonetheless possible to demonstrate the Doppler effect interferometrically using simple equipment. In this experiment, you will use a small He-Ne laser, an electronic spectrum analyzer, a photodiode detector, and some simple optical elements to measure the shift in frequency of light scattered from the back of a small moving train, a rotating disk, and from particles suspended in flowing water. From this information you will be able to determine the velocity profile of the water flowing through a tube, the speed of a train, the speed of a disk, and (after directly measuring the disk speed) the wavelength of the laser.

One disadvantage of the Doppler effect to measure speed is that you can only determine the velocity component along the line of sight. It is possible to measure the transverse speed optically in spite of this handicap, again interferometrically. The trick is to project first an interference pattern onto the object, then measure the frequency of the light fluctuations as it passes through the fringes. The fringe spacing is determined from the geometry of the experimental setup, and the speed is then given by the product of this spacing and the characteristic frequency of the scattered light fluctuations. For this part of the experiment a rotating disk will again be used, although a much faster rotation rate is required for the effect to be readily observable.

Theory and Concepts

At this point the question arises "How do we get around the problems mentioned in the introduction?" The answer is a technique known as *heterodyne interferometry*. This method involves combining the frequency-shifted or modulated optical (or radio, microwave, etc.) signal with an unshifted reference signal. The so-called "square-law" detector (which includes all common optical detectors) will respond only to the intensity (and intensity variation) of the combined beam. If the frequencies of the two signals differ by less than the bandwidth of the detector and are not identical, the electrical output of the detector will be modulated at a frequency equal to the difference between the frequencies of the two beams. For simplicity let us assume two electromagnetic waves of equal amplitude A and frequencies ω_1 and ω_2. If the two waves are incident on a detector we measure a signal

$$S = \langle (A\cos\omega_1 t + A\cos\omega_2 t)^2 \rangle \qquad (2.1)$$

where $\langle\rangle$ denotes averaging performed by the detector and subsequent electronics. Using the identity $2\cos\omega_1 t \cos\omega_2 t) = \cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t$ we can write the signal as

$$S = A^2\langle \cos^2\omega_1 t\rangle + A^2\langle \cos^2\omega_2 t\rangle + A^2\langle \cos(\omega_1 - \omega_2)t\rangle + A^2\langle \cos(\omega_1 + \omega_2)t\rangle \qquad (2.2)$$

Typically, in particular if we work with light, the frequencies $\omega_{1,2}$ cannot be resolved directly and the detection system averages the terms 1,2, and 4 in Eq. (2.2). The time dependence of the signal (the 3rd term in (2.2)) is then characterized by a frequency

$$\omega_{beat} = |\, \omega_1 - \omega_2|$$

This modulation is also referred to as a *beat frequency* or a *beat note* after the analogous acoustic phenomenon.**†** When two nearly identical tones are played together, the result is a `waa-waa-waa' variation of the loudness with a periodicity equal to the inverse of the frequency difference of the two tones.

So the heterodyne technique eliminates problem 1 above, as long as the Doppler shift is not too large. But what of problem 2? The stability of a small He-Ne laser, over a time span of a few seconds, is certainly no better than a several MHz. Thus you should not be able to measure speeds < 2 m/s ~ 10 km/hr, right? Wrong, as can be seen by comparing the coherence time of such a laser (tens of ns), the time delay between the signal and reference beams for the experimental arrangements used here (< 1 ns), and the Schawlow-Townes linewidth of a laser (~ Hertz). Qualitatively, what **would** be the limit of such a measurement? (Note: It is worth devoting a few lines to this in your report.)

The theory of the Doppler effect is covered in most basic physics texts, and will not be discussed here. The arrangement in the first part of this experiment is identical to a Michelson interferometer, and the expected frequency shift of the light can be derived by considering the object to be equivalent to the translating mirror. From this point of view, the beat note signal is just the rate at which fringes pass the detector. In the second (fast wheel) part, the measured frequency is the quotient of the transverse speed of the wheel and the fringe spacing, which is determined by the wavelength of the light used and the angle of intersection of the two beams. For the third (water flow) part of the experiment, the angle of intersection of the beams and the refractive index of water must be taken into consideration. The necessary derivations are left to the student and a summary thereof should be included in the lab write-up.

All three parts of this experiment involve measuring the light scattered from a large number of randomly placed scattering centers. Given that the characteristic frequency of interest is the same for all of the scatterers, the signal from each will be randomly phased so one might expect any modulation at that frequency to average out. Furthermore, since the collected light is due to scattering, the total intensity can vary wildly at frequencies unrelated to the one in which we are interested. Considering such a small signal to noise ratio, how can we expect to measure anything? (Hint: The net signal from an ensemble of randomly phased sources is a problem analogous to the drunkard's walk. Also, the beauty of using spectrum analysis is that it measures the total amplitude of signals over only a small range of frequencies. Thus the appropriate figure of merit is the ratio S/N where S is the signal amplitude at a frequency of interest and N is the amplitude of the noises whose frequencies are within one resolution bandwidth of the signal .

Experimental Procedures

This experiment is divided into three parts: a) Doppler shift measurement of the speed of a train and a rotating wheel; b) Transverse velocity measurement of the speed of a different wheel using scattering from a fringe pattern; c) Doppler shift measurement of the velocity profile of water flowing through a tube.

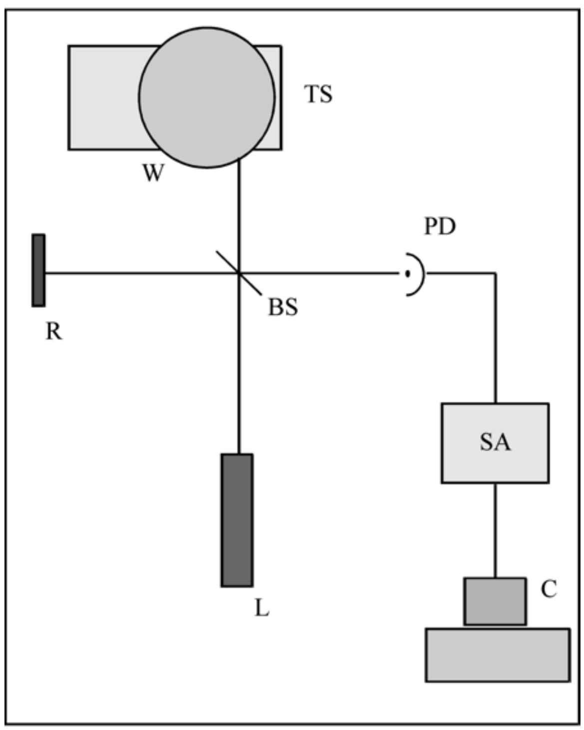

Figure 1. Experimental setup for the slow wheel Doppler Shift measurements. *L* is the laser source, *BS* is a beam splitter, *W* is a motor driven wheel with reflective tape on its rim, *R* is a reflector (also tape covered), and *PD* is the optical detector. The electrical signal is sent to the 3585A spectrum analyzer *SA* for analysis and then to the computer *C* for recording. The translation stage *TS* is provided for positioning the wheel.

Doppler shift from a train and a rotating wheel. The experimental setup for the wheel is sketched in Fig. 1. Note the equivalence of this arrangement to a Michelson interferometer. The beam from laser *L* is split by beam splitter *BS* and directed onto the edge of the slowly rotating wheel *W* and reference reflector *R*, both of which are covered with retroreflecting tape (of the type used by runners, bicyclists, etc., to avoid getting hit by cars). This tape is composed of small beads embedded in a substrate with a matched refractive index such that light entering the bead is refracted to the back of the bead where it is internally reflected back toward the source. This eliminates the need for the precise alignment required when flat mirrors are used. The retroreflected beams are then recombined at the beamsplitter and continue to detector *PD*. The electrical signal from *PD* is sent to a spectrum analyzer *SA* for measurement.

A spectrum analyzer (HP 3585A) is used to measure the beat frequency may be measured directly. You can transfer the spectra to a computer - see Appendix for instructions. First, calculate the expected Doppler shift from an estimate of the wheel rotation rate and set the analyzer accordingly. Before connecting the detector signal to the spectrum analyzer, look at it with an oscilloscope; what is the amplitude of the signal, and does its period correspond to your calculated estimate? Measurements are to be made over a range of wheel positions at roughly one centimeter intervals, for which translation stage *TS* is provided. The range chosen should be such that one edge is accessible. Note: You must be sure that you are looking at the right peak on the spectrum analyzer trace. Determine this by blocking either the reference or shifted beams (the signal will then disappear) and by translating the wheel (the signal frequency should then shift). Once the data are taken, you will also need the diameter of the wheel, a direct measurement of its rotation rate, and the wavelength of the laser. From these you should be able to determine the rotation rate of the wheel and to estimate the precision of your measurements. Discuss the precision of your results and the important error sources.

Transverse speed measurement. As pointed out earlier, the Doppler technique is applicable only for measuring longitudinal velocities. (There is in fact a transverse Doppler effect, but it goes as v^2/c^2 and so is observable only for relativistic particles.) However, it is possible to measure the transverse velocity of an object using light scattering. The setup is as shown in Fig. 2. The laser beam is split and reflected by *BS* and mirror *M* onto the edge of the fast rotating wheel *W*, intersecting at its surface at an angle a to form a fringe pattern of spacing δL. This spacing should be too small to be visible with the unaided eye but much greater than the wavelength of light (i.e., between 5 and 50 μm). As it is determined by the beam crossing angle α, you must position *BS* and *M* appropriately. The most accurate way to find this angle is to measure the separation of the beams some distance from their intersection; take several such measurements at different positions to obtain an accurate value for α.

The scattered light may be viewed from the side, as shown in Fig. 2. A lens *L* is used to collect the scattered light and focus it onto the detector *PD*. As before, the detector signal is sent to the spectrum analyzer. The signal frequency of interest may be discerned by blocking either of the interfering beams - it should disappear. Position the wheel transversely to maximize the scattering frequency (i.e., so that the scattering comes from the foremost point on the wheel). Adjust the spectrum analyzer to fully resolve the signal frequency and measure its value. Also measure the diameter of the wheel and estimate its rotation rate observing the detector signal on an oscilloscope - though it may be quite noisy, it should be periodic over one rotation.

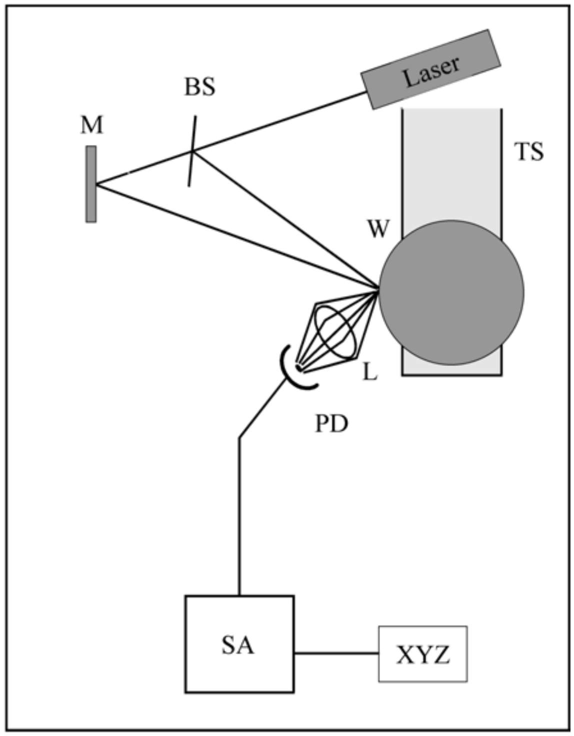

Figure 2. Experimental setup for the transverse speed measurements. Most labeled parts are as in Fig. 1, except that *L* is a collecting lens (optional) *M* is a flat mirror, and *XYZ* is a PC to which the spectra can be transferred.}

Report the measured frequency and its bandwidth. Calculate α and δL, and the rotation rate. How does this compare with the oscilloscope estimate? Which would you expect to be more accurate and why? What are the major sources of error in this experiment, and in particular what is the effect of the bandwidth of the measured scattering frequency?

Velocity profile of flowing water. This is probably the most difficult part of the experiment. It is diagrammed in Fig. 3. A small portion of the laser beam is split off by the microscope slide *BS* (reflectance ~ 4-5% per face). This reference beam is directed through the flow tube *FT* perpendicular to its faces and through aperture *A* and lens *L* onto detector *PD*. The rest of the laser beam is reflected at mirror *M* to intersect the reference beam at a point inside *FT*, at an angle of 10^o – 35^o. (This angle is measured as in the transverse velocity measurements above - but be sure to account for the differing indices of water and air.) The tube is filled with water in which a colloidal substance has been suspended - CoffeeMate nondairy creamer works nicely - and is connected to a pump. The suspended particles serve to scatter some of the strong signal beam into the detector where it will be mixed with the reference, any frequency shift will then be observable with the spectrum analyzer. Note the similarity of this arrangement to a Mach-Zehnder interferometer. We should expect the signal beam to be Doppler shifted by an amount determined by the projection of the flow velocity onto the direction of the signal beam (the derivation of the expected shift is left to the student).

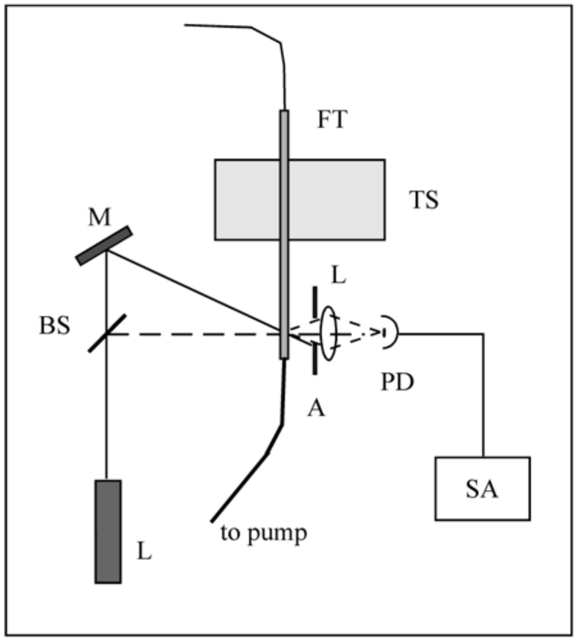

Figure 3. Experimental setup for the flow profile measurements. *BS* is a microscope slide, *FT* is the flow tube, and *A* is an aperture.

Several important factors must be considered for this experiment to work properly. First, it is essential that the flow be laminar - a turbulent flow will give Doppler shifts over a wide range of frequencies, blurring the spectrum analyzer peak until it becomes unrecognizable. Second, we must limit the size of the particular scattering region at which we are looking for each measurement - if the volume is large, there will again be a range of frequencies in the spectrum. This is the function of the lens: it images a small region within the intersection volume onto the detector. Third, we must restrict the range of scattered wave vectors - light scattered off at different angles will have different Doppler shifts, again broadening the frequency peak. This is the function of the aperture *A*: although the lens will image any light scattered from the volume of interest onto the detector, the aperture blocks all but that light propagating along the direction of the reference beam. (A vertical slit could be used instead. Why?)

Special care must be taken to obtain a signal which is sufficiently clean for taking measurements. The detector should be placed so that it receives the reference beam directly. The position of the lens is particularly critical. Initially determine the approximate height and transverse position by centering the lens on the reference. Then find the longitudinal position in the following manner: Translate the tube so that the beams intersect at the face nearest the detector. Block the beams with a thin sheet of paper and adjust the positions of *L* and *PD* to image the beam-spot onto the detector element. Then remove the paper and check that the reference beam is not deflected away from the detector - if it is, repeat the positioning steps. When this procedure is completed, translate the flow tube *FT* so that the beam intersection is in the middle of the tube. Adjust the spectrum analyzer until a signal is found (start by estimating the expected Doppler shift). The true beat note signal will disappear when either of the beams is blocked. Check to see that the frequency varies with the pump speed. When you have found the beat note, adjust the position of the various components to maximize the signal amplitude. If the signal is sufficiently above the background (> 10 dB), you will be able to take the necessary data by computer as with the slow wheel. Take measurements at ~1 mm intervals starting at one side and continuing until the other side is reached. In analyzing your data, plot the profile and qualitatively compare it with what would be expected for a laminar flow (look it up). As usual discuss experimental problems, error sources, etc. Note: On occasion this experiment does not work out very well. If such is the case for you, at least obtain a measurement of the speed at the center of the tube and explain the problems encountered.

Summary of Procedures

Upon completion of this experiment you should have sufficient data to obtain the following:

- The rotation rate of the slow wheel using the Doppler effect.
- The speed of the toy train (compare with direct measurement).
- The rotation rate of the fast wheel using the interference-scattering technique.
- The velocity profile of a laminar flow, via the Doppler effect. \end{itemize}.

Food for thought

Doppler velocity with the Michelson arrangement. It is not really necessary to invoke the Doppler Effect. Find an alternate explanation (it should lead to the same numerical expression)

Transverse velocimetry measurement (fast wheel). This can be interpreted also through the Doppler effect…

REFERENCES

1. M. V. Klein and T. E. Furtak, *Optics* (Wiley, New York, 1986).

APPENDIX: HP 3585A Spectrum Analyzer

The Hewlett-Packard 3585A is a digital spectrum analyzer with a bandwidth of 20 Hz - 40 MHz. It can be operated manually by the front panel controls, or by computer via the IEEE-488 interface. The manual method is the subject of this appendix.

The entry keys of the front panel are divided into eight groups: INPUT, ENTRY, SWEEP, TRACE, MARKER/CONTINUOUS ENTRY, RBW-VBW-ST, STATUS, and TRIGGER. The pertinent keys in each of these groups will be discussed below. Most keys are designed so that they will be lit, or a corresponding entry on the CRT display highlighted, when on or in effect.

Before discussing the key panel, look at the CRT display. The horizontal axis is in units of frequency, and either the START and STOP frequencies or the CENTER frequency and frequency SPAN will be given below the bottom grid line. The vertical scale gives the power of the signal in logarithmic units (dBm, dBV, or dB) with the power corresponding to the top grid line given by REF, the values for the other lines decreasing from this value by the amount given in dB/DIV, both displayed in the upper left corner. (Remember that a signal ratio of 10 dB implies an electrical power ratio of 10, while a ratio of 20 dB implies a {\it voltage} ratio of 10. For most optical detectors, it is the output voltage that is proportional to the optical power in.) The RANGE determines the noise level due to the internal amplifiers. The upper right corner usually gives the MARKER position, but may display other information, and will be discussed later. The meaning of the entries on the bottom line will likewise be discussed below. The CRT is also used to display various messages, which will generally be accompanied by a warning tone. For example, every two minutes or so the instrument will execute a self-calibration routine and ``CALIBRATING" will briefly appear on the screen; similarly, if front panel operation is attempted while the device is in the remote mode, the message ``HP-IB OPERATION ONLY" will be displayed.

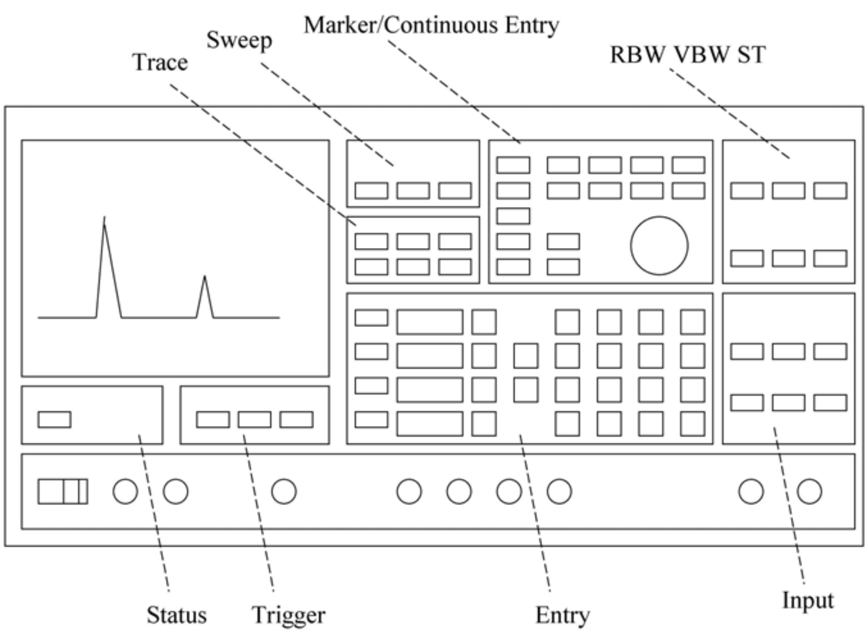

Figure 1. Layout of the HP 3585A Spectrum Analyzer front panel

The Eight Panel Groups

1. *INPUT*. Proper settings and connections of the INPUT group are required to obtain a trace, and to prevent overloading. For low frequency applications, the IMPEDANCE should be set to 1~MΩ; the 50 Ω and 75 Ω inputs need to be used only when investigating frequencies of >10 MHz and when the source is capable of driving such an impedance. The signal source must be connected to the input jack corresponding to the active IMPEDANCE setting. When AUTORANGE is on, the input amplifier RANGE will be adjusted to suit the input signal; with REF LEV TRACK on the power level corresponding to the top of the display grid will follow any adjustments of the RANGE (whether this is due to AUTORANGE, or to any manual STEP KEY or numerical entry)

2. *ENTRY*. This group is the most important of the eight. In particular, the REFERENCE LEVEL, dB/DIV, START FREQ/STOP FREQ and/or CENTER FREQUENCY/FREQUENCY SPAn may be set using the STEP, numerical, and unit keys. To change the value of any of these parameters, press the corresponding key. The corresponding CRT entry will then be highlighted, and the value may be changed either incrementally using the STEP keys (which usually change the parameter by a factor of 2 or in a 1-2-5-10 or 1-3-10 sequence) or by numerical entry from the keypad. If the new value is not allowed a bell will sound and possibly an error message will be displayed; the parameter will remain highlighted awaiting an acceptable entry. A change implemented by the STEP keys will be effected immediately, while a numerical entry will not take effect until an appropriate unit is entered. In either case, the CRT entry will remain highlighted until a units key has been pressed.

3. *SWEEP*. The SWEEP (i.e., the scanning of the internal reference oscillator through the appropriate range of frequencies) may be either CONTinuous (a new sweep beginning as soon as the last one is completed) or SINGLE (sweeping stops when the STOP FREQ is reached). In either case the SWEEPING light will be lit and the CRT trace will be continually updated while the sweep is in progress. The MANUAL ENTRY mode is rarely useful.

4. *TRACE*. The 3585A can store two traces: A, which is continually updated during sweeping, and B, which can be used for permanent storage. A trace must first be obtained in the A trace, then it can be transferred to B by pressing the STORE A→B key. Either or both can be viewed at any time, as can their algebraic difference A - B, by activating the appropriate key. The B trace is particularly useful for storing a good trace for later plotting, or for storing baseline noise trace which can then be subtracted from the A signal trace.

5. *MARKER/CONTINUOUS ENTRY*. The MARKER is the highlighted pixel on the spectrum trace which can be positioned by the CONTINUOUS ENTRY knob. The frequency and signal amplitude at its position on the trace will be given in the upper right corner of the CRT when it is active. When the COUNTER feature is active the trace will stop for $\approx$ 0.1 s when the marker frequency is reached and the device will measure the frequency (to an accuracy of 0.1 Hz) of the strongest signal within one resolution bandwidth (RBW - see below) of the marker frequency. The other features in this group are of little interest for our purposes.

6. *RBW-VBW-ST*. The RESolution Bandwidth is the frequency range allowed through the input filters to the system amplifiers, it is thus the displayed width of any input signal of bandwidth less than the RBW value displayed at the lower left corner of the CRT. It can be reduced to

resolve closely spaced signal or increased to speed up the sweep rate if the extra resolution is not needed. The VIDEO BandWidth (VBW - bottom center of the CRT) is, qualitatively speaking, the inverse of the time over which a particular element of the display is average; as such, it can be reduced to average fluctuations in the noise level which might mask weak signals, or increased to reduce the SWEEP TIME. With the COUPLES TO SPAN feature active, the SWEEP TIME (ST - lower right corner of the display) will be automatically set to the minimum value for which the instrument can still take calibrated measurements; if the ST is too small for the RBW and VBW, the UNCAL LED will light, and the SWEEP TIME increased until it goes off. The RBW and VBW can be varied in a 1-3-10 sequence only; the ST can be set to any multiple of 0.2 s.

7. *STATUS*. When the 3585A is being controlled via the IEEE-488 [also know as the HP-IB (Hewlett-Packard Interface Bus) and GPIB (General Purpose Interface Bus)] interface on the rear of the instrument, the REMOTE LED will be lit and the front panel will be disabled. The other three LEDs indicate the status of the instrument with respect to the bus. REMOTE control may be overridden and front panel operation restored by pressing the local key.

8. *TRIGGER*. The FREE RUN mode should be used.

Plotting. Though not an entry key group, the capability to generate a hard copy plot of the displayed frequency spectrum is a useful feature of the 3585A. To obtain a plot, an X-Y recorder must be connected to the X and Y PLOTTER OUTPUT connectors on the back panel of the instrument (if the recorder provides a remote pen lift feature, this should be connected to the Z output - note however that there may be some compatibility problems). The plot is initiated by pressing the RECALL (on) key [lower left corner of the ENTRY block] then the 8 (plot1) key. An analog copy of the trace will be sent to the output jacks at a rate which a typical recorder will be able to follow. Output voltages will range from 0 V (START FREQUENCY) to +10 V (STOP FREQUENCY) for X, and from 0 V (lower grid line) to +10.64 (REFERENCE LEVEL = 10.4 V) for Y. Set the recorder inputs ranges accordingly. The outputs are set to the high limits after each plot, so zero the recorder at the upper right corner of the page.

A simple demonstration - ``Watching the radio". To demonstrate the operation of the 3585A, connect a short piece of unshielded cable to the 1MΩ input. Tune the START and STOP frequencies to locate the AM radio band (0.5 - 1.6 MHz). Position the marker on one of the stations and measure the frequency of the station using the COUNTER. Watch the frequency and amplitude of the signal over several measurements. Can you see why these are `amplitude modulation' stations rather than `frequency modulation'? Can you find the FM stations?

As mentioned in the former subsection, the Hewlett-Packard 3585A spectrum analyzer may be remotely operated by computer via the IEEE-488 interface [also known as the HP-IB (Hewlett-Packard Interface Bus, or GPIB (General Purpose Interface Bus)]. The program DOPPLER.EXE has been written to drive the 3585A and to collect the appropriate data for this experiment. Use of this program is the subject of this portion. Throughout this section anything that is enclosed in quotes “ “) is what you will be expected to type from the keyboard; names of special keys which you must type will be enclosed in Dirac brackets (e.g., <esc>, <enter>, etc.).

Setting up. Turn on the computer, and the 3585A. Get into the optics lab directory by typing “CD/OPTLAB<enter>". Obtain a signal trace on the analyzer and move the marker to the top of the peak. Once this is done you are ready to initiate the program by typing “DOPPLER<enter>".

Initializing the program. The program will begin by displaying a summary of its operation, which will remain on screen until you press <enter>. The screen will then be cleared and you will be asked for the name of a file in which to store your data. Enter this as a root name of up to eight letters followed by a period followed by a three letter suffix (########.###, e.g., MYDATA.NEW or GEORGE.DAT). The computer will then take control of the 3585A and take a datum as an operational test; so long as the system does not hang up and no bells are sounded, everything is OK. You will then be asked to type ``T<enter>" when you are ready to begin data taking or ``A<enter>" if for some reason you wish to abort the program.

Data collection. You will next be prompted for the initial position reading of the translation stage. Enter this in centimeters or millimeters, but always be consistent - use one or the other throughout the session. The computer will then turn the 3585A COUNTER feature on, trigger a trace, wait for the analyzer to measure the frequency, then request the value of the measurement. It will then display (on the computer CRT) the number of data points taken thus far, the current position of the translation stage, and the current frequency measurement. It will also move the spectrum analyzer MARKER to the measured frequency so that you can determine whether the counted frequency corresponds to the Doppler shift signal. You will then be asked whether to keep the datum or reject it. The datum should be rejected if the reported does not correspond to the reading from the translation stage (in which case you moved to the peak of the Doppler shift signal (in which case you should move the MARKER manually to that position using the CONTINUOUS ENTRY knob on the 3585A front panel - this will prepare the instrument for the next measurement). After keeping/discarding the datum you will be prompted with “Type <enter> to continue, <space><enter> to change position, or STOP to quit”. Respond accordingly. If the first option is chosen, a new trace will be triggered and a new measurement taken. If you want to ‘change position’ you will be prompted for the new position; move the translator, type in the new reading, and press <enter> to indicate a new measurement. Respond with “STOP” if you are finished taking data or if you wish to quit the session; the output data file will be closed and the 3585A will be returned to manual control.

Suggestions. You will want to take several data points at each position to get an idea of the accuracy of your measurements. Also, you may need to change the CENTER/SPAN or START/STOP frequencies to keep the signal on the trace. Similarly, the RBW, VBW, and dB/DIV may need to be adjusted to suit the signal. If you have problems with spurious signals, the A-B feature is very useful.