# Holography

# 1 Background

The word hologram was coined from the Greek word "holos" which means whole, because the recorded hologram contains all the information of the image field . Unlike a photograph, which records the square of the electric field, a hologram contains both the amplitude and phase information from the image. Although originally invented by Dennis Gabor in 1948 for the purpose of improving electron microscope images, holography did not attract much interest until the discovery of the laser in 1960 [1]. Since then, both artistic and scientific communities have explored the field enthusiastically, finding numerous applications in holographic optical elements (HOE's), optical storage, imaging, and of course, visual arts.

## 1.1 On-axis holography

In Gabor's original work, two waves are superposed on a recording medium as shown in Fig. 1.



Figure 1: Gabor's scheme for recording a hologram. (O1=Reference wave; O2=Object wave)

A small semi-transparent object is illuminated by a light source. The object scatters the light and creates a second wave which superposes on the recording medium with the reference wave originating from the source. The field on the recording plate can be written as  $\tilde{E}_{total} = \tilde{E}_r + \tilde{E}_o$ where  $\tilde{E}_r$  is the electric field of the reference wave and  $\tilde{E}_o$  the field of the scattered or object wave. In terms of amplitudes and phases,  $\tilde{E}_{total}$  can be represented in the following form:

$$\tilde{E}_{total} = e^{i\psi_r(x,y) - i\omega t} \left[ E_r + E_o e^{i(\psi_o(x,y) - \psi_r(x,y))} \right].$$
(1)

Thus the recorded intensity is

$$I = |E_{total}|^2 = \left[E_r^2 + E_o^2 + E_r E_o e^{i(\psi_o(x,y) - \psi_r(x,y))} + E_r E_o e^{-i(\psi_o(x,y) - \psi_r(x,y))}\right].$$
 (2)

If the exposure time is T, the amplitude transmittance  $\tilde{t}$  of the developed plate is  $\tilde{t} = \tilde{t}_0 - \beta T I$ . Here  $t_0$  is the amplitude transmission of the unexposed film. The term  $E_o^2$  in Eq. (2) contributes to a noise in the reconstruction. This noise is small compared to the reference background  $E_r^2$ . If we remove the object and illuminate the developed plate with the original reference beam, the transmitted field will be:

$$\tilde{E}_t = E_r e^{i\psi_r(x,y) - i\omega t} \tilde{t}.$$
(3)

The field  $\tilde{E}_t$  is proportional to:

$$e^{i\psi_r} \left[ E_r + E_o e^{i(\psi_o(x,y) - \psi_r(x,y))} + E_o e^{-i(\psi_o(x,y) - \psi_r(x,y))} \right].$$
(4)

We recognize in the first term the wave which originally illuminated the plate; the second term is identical to the wave scattered by the object. Such a plate which has the property of keeping the amplitude and phase of a wave is called a hologram. In Eq. (4), the third term is identical to the second except for the phase term  $(\psi_o(x, y) - \psi_r(x, y))$  which corresponds to a second reconstruction, located on the opposite side of the plate. This conjugate wave which appears during reconstruction, is not seen when we focus on the primary reconstruction. However, its presence degrades the primary wave and prevents getting a high quality hologram. More complete discussions of the theory are in references [1, 2, 3].

#### 1.2 Off-axis holography

The biggest challenge of the on-axis holography is the twin-images: a conjugate image locating right in front of the true image. Instead of sending the reference beam and object beam in line, Leith and Upatnieks proposed separating and sending them at different angles. However, to separate the beam was difficult in practice at that time, when the best coherent source was the high pressure mercury lamp, with coherent length of only about 0.1mm. The first off-axis hologram worked until Leith and Upatnieks came up an optical trick: to choose one line of the mercury lamp and send it to a grating. Two different orders of the light after the grating are used as the reference and object beam, which are automatically propagating at different angles. The technique of holography was greatly improved after 1961 when laser was used as the light source. The experimental set-up for off axis holography is shown in Fig. 2. The beam emitted by a laser is split into two beams. One beam is expanded and sent onto a high resolution recording medium. The second beam is used to illuminate an object. The light scattered by the object interferes in a complex fashion with the reference light. The interferences are recorded on a photosensitive plate. After development of the plate, an object wave can be reconstructed by illuminating the plate with the reference wave at the same angle as for the recording (shown in Fig. 3). With this off-axis hologram it is possible to detect the object wave without being disturbed by the reference beam which propagates along another direction. Off-axis holography requires the use of a laser for coherence requirement.

#### 1.3 Transmission and reflection holograms

The fundamental difference between a transmission hologram and a reflection hologram lies in the direction of the interference fringes that are recored inside the photosensitive emulsion. In a transmission hologram, as shown in Fig. 4 (a), the reference wave and object wave entering the emulsion from the same side produce interference fringes in planes that are perpendicular to the plane of the emulsion; while in a reflection hologram, as shown in Fig. 4 (b), the reference wave and object wave entering the emulsion from different sides produce interference fringes in planes that are parallel to the plane of the emulsion. Observation of the image in a transmission hologram requires the same reference beam that exposed it, and viewing through the plate; while in a reflection hologram, a spot light or sunlight is good enough (why?) and the image is



Figure 2: Experimental setup for off-axis holography. (M=mirror; BS=beam splitter)



**Figure 3:** Setup for reconstructing the image using the same reference beam as in the expose setup. The reference beam is diffracted by the recorded fringe patterns on the plate in a way such that the diffracted beam seems to come from where exactly the original object locates.

## a) Transmission hologram b) Reflection hologram



Figure 4: Top view of fringes creation in a transmission and a reflection hologram

observed by viewing the reflection from the plate. The holographic plates you are going to use are good for recording both transmission and reflection holograms and you can use both setups in this experiment.

#### 1.4 Amplitude hologram and phase hologram

No material can record both the phase and amplitude of the electric field. The idea of a holography is to encode the phase information into intensity variation. In an amplitude hologram, the phase information is recorded as transmission variation (optical density variation). When the Silver-Halide film is exposed to light and developed, the grains of silver halides are changed into metallic silver and the transmittance of the film is altered. The amplitude transmittance  $\tilde{t}(x, y) = t_0 - \beta I(x, y)T$  varies spatially the same way as the exposure intensity.

In a phase hologram, the phase information is recorded as index variation. Thus the amplitude transmittance has magnitude equals to 1 and a varying phase  $\tilde{t}(x,y) = e^{-i\Phi(x,y)}$ . One can convert an amplitude hologram into a phase hologram by bleaching. Since a phase hologram has much higher diffraction efficiency than an amplitude hologram, it's always more preferable in experiments. All the holograms you are going to make in this lab are phase holograms.

## 2 Holographic plates data sheet

The first requirement for a good hologram is a film with high enough resolution to record the pattern created by the interference of the reference and object beam. You will use Slavich PFG-01 which can resolve more than 3000 lines per millimeter.

The curve on the right is for PFG-01. The spectral sensitivity range 600-680 nm, average grain size is 40 nm, and emulsion thickness 7  $\mu$ m. The diffraction efficiency is larger than 40% at appropriate exposure energy. The second curve (Optical density v.s. exposure energy or H-D curve) in Fig.5 shows the optical density response v.s. average exposure information that you need to calculate the exposure time and the reference/object beam intensity ratio (K) when you are using two beam set-up. You can see from H-D curve that the response of the the film is not linear over the whole exposure range. If the light intensity modulation falls in the nonlinear region of the H-D curve, it will bring noise and distort the image; however, if K is very large, it has a good linear response but the modulation depth may be too small to have high diffraction



**Figure 5:** Characteristic curves for PFG-01 holographic plate, from Integraf LLC. Density=Optical density, transmission $|\tilde{t}|^2 = e^{-D}$ . It's plot in logarithm because the human eyes' response to light intensity is in logarithm. Diffraction efficiency= diffracted intensity at desired order/ incident intensity of the reconstruction beam

efficiency, making a very dim image. There is trade-off in having a linear response and high diffraction efficiency. K being from 4 to 10 (depending on the reflectivity of the object) are reasonable numbers. You can find more information about the characteristics of Silver-Halide in the book [4].

## 3 Holographic plates development

Developing the film is the second critical step, and a myriad of developing techniques and recipes exist. The following one is that of T.H. Jeong from Lake Forest College. The Part A solution, Part B solution and Bleach solution are prepared from the JD-2 chemical processing kit and they are prepared for you before the lab. The following procedures will be carried on in a separated dark room.

## Preparation

Have the following items on hand:

- Pre-mixed Part A, Part B and Bleach solution.
- 1 additional gallon of distilled or de-ionized water for best results. Tap water may also work, but not as well. Avoid hard water. All in room temperature.
- 2 small plastic trays and 2 large trays.
- 1 rubber glove

Label one small tray as Developer A & B. Then mix equal portions of Part A and Part B, enough so that the hologram to developed can be totally submerged. Once mixed, the combined A& B can be used to develop several holograms and lasts several hours.

Next to the developer tray place a large tray with half liter of distilled water. This will be used as a rinse.

Next, label another small tray as Bleach. Put enough bleach into it so that the hologram can be totally submerged. The bleach can be returned to its original container and re-used for many times.

Next to the bleach tray place another large tray with half liter of distilled water. This will be used as a rinse.

Check to order of the trays: **Developer A** &  $\mathbf{B} \longrightarrow \mathbf{Rinse} \longrightarrow \mathbf{Bleach} \longrightarrow \mathbf{Rinse}$ 

## Process procedures

After exposing the plate, keep track of the emulsion side and always keep the emulsion side up to avoid scratched from the developer tray.

- 1. Develop: Quickly submerge the plate into the developer so that all parts get wet evenly. Slush it around for about two minutes. The hologram should turn almost black.
- 2. Rinse: Rinse the developed hologram with agitation for at least 20 seconds. For best results and longer lasting holograms, rinse up to three minutes to make sure that all of Part A has been rinsed off.
- 3. Bleach: Place the rinsed hologram into the bleaching solution; agitate it until the plate is completely clear (this may take up to two minutes); then, bleach for another 10 seconds.
- 4. Rinse again: Rinse the bleached hologram with agitation for at least 20 seconds (up to 3 minutes).
- 5. Dry: Stand the plate vertically with a paper towel under it, dry in room temperature.

## 4 Experiments

## 4.1 Setup consideration

## 4.1.1 Spatial Filtering

Practice aligning a spatial filter to make a clean beam.

## 4.1.2 Coherence and stability measurement

Since a hologram is recording INTERFERENCE fringes, both the coherence length of the laser, and the stability of all the optics you are using, play a crucial role in creating stable interference fringes. Use a expanded collimated laser beam to set up a Michelson interferometer. Change the optical path length difference between two arms and observe the fringe contrast and stability. This practice will allow you to have some idea of how sensitive the interference fringes are to optics stability, and how much optical path length difference is allowed to still create high contrast fringes.

#### 4.1.3 Polarization

Polarization is important in building an interferometer. Although it is not as critical in a holography experiment, it is still helpful to maintain the laser beam at the same hight after each optics to keep the same polarization.

The holographic plates that you will use are not anti-reflection coated, there's always some multiple reflection between two surfaces of the plates, creating interference fringes on the emulsion. They are the so called "woodgrain". One way to minimize this effect is to manipulate the laser polarization such that it is parallel to the plane of incident (Ppolarized). The result will be best if incident at Brewster's angle since no P-polarized light will be reflected at Brewster's angle. But P-polarized light is probably enough because it is reflected less than s-polarized light (see any optics text book). Check the polarization of the laser and rotate it if necessary using a half-wave plate.

#### 4.2 Simple hologram

Now you've been introduced to different configurations of holograms (on-axis, off-axis, reflection, transmission). Use one configuration you like to produce a hologram of a small rigid object (coins for example) and read out the image after development.

## 4.3 Holographic grating

Holographic gratings are made by recording the interference intensity patten of two plane waves onto a light sensitive material. Holographic gratings are much more popular than conventional rule gratings nowadays because they are easy to make at low cost; they have smaller scattering loss; can be made much higher number of grooves per mm for high resolution requirement and of large area.

The beam intensity ratio consideration in making a grating is much simpler because there is no intermodulation among the scattered object beams. Thus the dynamic range of the holographic plate is usually larger than what is stated in the data sheet from the manufacturer. The beam ratio can be 1:1 and average exposure energy can be twice as recommended in the data sheet.

To better understand this experiment, we start with introducing a Q factor that defines two different kinds of holographic gratings—thin (plane) grating and thick (volume) grating. In a thin grating(periodic lines structure), each line acts as an individual scattering center and there is no "talking" between them. The far field diffraction pattern is formed by the superposition of each scattering wavelet. While in a thick grating (periodic planes structure), the incident wave and diffracted wave are coupled together as they propagate along the thick periodic structure and there's energy transfer between them. In this case, strong diffraction can only be seen when the Bragg condition is met.

The Q factor is defined as:

$$Q = 2\pi \frac{\lambda/n}{\Lambda} d/\Lambda \tag{5}$$

In which  $\lambda$  is the incident light wavelength in vacuum, n is the refractive index of the emulsion,  $\Lambda$  is the grating period, d is the emulsion thickness. A grating is considered to be thin when  $Q \leq 1$ , thick when  $Q \geq 10$ . Derive the phase matching conditions for both



a) Thin transmission grating b) Thick transmission grating

Figure 6: Thin and Thick grating with the same emulsion thickness

cases in your lab report. With a 7  $\mu m$  thick emulsion, you could make both thin and thick gratings.

## 4.3.1 Make A Thin Phase Grating

As discussed in the introduction, the transmittance function of a lossless phase grating can be expressed as  $\tilde{t}(x, y) = e^{-i\Phi(x,y)}$ . Assume that the phase modulation is only along x axis and the phase changes are linear to the exposure intensity changes. Then,

$$\Phi(x) = \phi_0 + \phi_1 \cos\left(Kx\right) \tag{6}$$

Where K is the grating vector and  $K = \frac{2\pi}{\Lambda}$ ;  $\phi_1$  represents the phase modulation depth. If we ignore the the constant phase term  $\phi_0$ 

$$\tilde{t}(x) = e^{-i\phi_1 \cos\left(Kx\right)} \tag{7}$$

You can see from Eq 4.3.1 that it is not a simple form as  $e^{-iKx}$  (representing a single spatial frequency, but rather contains a large number of spatial frequency if we expand it as Fourier series:

$$\tilde{t}(x) = \sum_{n=-\infty}^{\infty} i^n J_n(\phi_1) e^{-inKx}$$
(8)

Unlike the amplitude grating, the sinusoidal modulation on phase leads to multiple orders of spatial frequency. This phase grating can then diffracts the incident light into multiple orders. What is the theoretical maximum diffraction efficiency to the  $1^{st}$  order?

Design your set up to make a thin phase grating with Q = 0.1. If an unexpanded laser beam is impinging on this grating with zero degree incident angle, how many diffraction orders do you expect to see? At what angles do you expect to see them? Suppose you are making a sinusoidal phase grating, if the maximum index modulation  $\Delta n = 0.05$ , what is the amount of power do you expect to be diffracted into these orders? Measure the diffraction efficiency of each order you observe and calculate the index modulation they correspond to. Is your number reasonable, considering the index modulation is usually from 0.02 to 0.1? What might the error come from? Measure the diffracted angles of each order and compare them to the theoretical values. Why are they usually larger than theory?

#### 4.3.2 Make A Thick Phase Grating

For more information about the coupled wave theory, please refer to any holography text book or ask for a copy of the article [5]. Design your set up to make a thick grating with Q = 100. What's the number of grooves per mm in this case? Can it be resolved by the PFG-01? How many diffraction orders do you expect to see? Observe the diffracted beam as you tune the angle of incidence. Measure and plot the the 1<sup>st</sup> order diffraction efficiency as the deviation angle ( $\Delta \theta$ ) from Bragg condition and discuss the angular selectivity of this thick grating. Now you can use the grating you make to analyze the spectrum of white light source, or use it as a adjustable beam splitter!

#### 4.4 Holographic interferometry

Because holography offers the unique potential to capture the complete electrical field, it is possible to interfere two reconstructed images with each other. This is what is done in a double exposure hologram. In this lab a single exposure will first be taken of an aluminum bar. Then without moving anything, a small weight (5-10g) will be added to the bar and a second exposure will be made. Since the phase information from both exposures is stored in the hologram, any changes in phase introduced by a small bending (on the order of microns) of the bar will show up as fringes in the reconstructed image. A possible setup for making such a double exposure hologram is shown in Fig. 7, and more information can be found in the references [6].

Upon viewing the developed hologram, one should see a series of fringes crossing the bar (fringes perpendicular to the length of the bar). As might be expected, dark fringes occur where the change in optical path length between the two exposures is equal  $n\lambda/2$ , where n is an odd integer. If one can assume that for each point along the length of the bar, the displacement due to the extra mass is approximately perpendicular to the surface of the bar, then it is possible to show (you should do this for your report) that fringes will be observed when the displacement  $\Delta y$  is given by

$$\Delta y = \frac{n\lambda}{2(\cos\alpha + \cos\beta)}.\tag{9}$$

Here  $\alpha$  and  $\beta$  are the incident angle of the laser light and the angle of observation respectively, both of which are measured from the normal of the surface of the bar.

The setup in 7 is a on-axis reflection hologram setup. Because of the limited coherence length (you should have some idea of that when you test it with a Michelson interferometer) of the laser, you should place the bar as close to the film as possible. Or you can also build a off-axis setup allowing you to have exact control of the optical path length difference. Give the entire apparatus some time (2 min) to "settle." This will allow any small vibrations to damp out. When all is calm, expose the film. Then quickly remove the weight from the bar, wait a few seconds for vibrations to settle and expose the film again for the same period of time.



Figure 7: Apparatus for a double exposure hologram. m1, m2, m3 are mirrors.



**Figure 8:** The displacement of the bar due to the additional weight changes the optical path length from ABC to DEF. At each small segment of the bar, the displacement is assumed to be perpendicular to the surface of the bar.

After developing the film, you will want to carefully measure the spacing between the fringes and use Eq. (9) to calculate the bending of the bar along its length. Compare your results with the theoretical predictions for the bending of a rigid steel bar due to a force at one end. A description of this situation can be found, for example, in Stephenson's book [7].

## 4.5 Additional Experiments (optional)

If you have enjoyed your holographic experiment thus far, you may want to continue by making a double exposure hologram of another stressed object. An aluminum can may easily be deformed between exposures by tightening a screw against its side, for example. Another type of experiment involves studying the normal modes of a vibrating object (a handbell driven with a small electromagnet would be a good candidate). In this case only one exposure is necessary, and even though the object is moving during the exposure fringes are still observed. This is because on average the oscillating surface spends most of its time at the two turning points. One then obtains the equivalent of a hologram with an exposure taken at each extremum. Unfortunately, with the more complex geometry an exact analysis of the resulting fringe pattern is not always possible.

Another interesting experiment is to produce a Fresnel lens. You can use the same setup of section 4.3. Place a lens in one arm so that in the film plane you have a superposition of a parallel and a converging beam. Readout the developed hologram with the expanded parallel beam and search for the focus spot produced by the lens. Measure the focal length and explain.

## 5 Summary

- 1. Produce a simple hologram of a small object.
- 2. Produce two holographic gratings. Measure and discuss the diffraction efficiency and the diffraction angle.
- 3. Investigate the bending of a metal bar under the influence of a weight using holographic interferometry. Compare your results qualitatively to theory. Plot a graph  $\Delta y(z)$  comparing experiment and theory.
- 4. Produce a hologram of a vibrating object. (optional)

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